1: Consider following system of equations:

\[ 4x_1 + 3x_2 + 6x_3 = 6 \]
\[ 2x_1 + 6x_2 + 2x_3 = 4 \]

(a) Write this in matrix form, \( Ax = y \).

\[
A = \begin{pmatrix} 4 & 3 & 6 \\ 2 & 6 & 2 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad y = \begin{pmatrix} 6 \\ 4 \end{pmatrix},
\]

(b) Compute a generalized inverse \( A^{-} \) of \( A \).

```r
In R
> library(MASS)
> A <- matrix(c(4,2,3,6,6,2), nrow=2)
> gA <- ginv(A)
> gA
```

```
[,1]     [,2]
[1,] 0.08064516 -0.02419355
[2,] -0.07741935  0.20322581
[3,]  0.15161290 -0.08548387
```

(c) Recalling that a g-inverse satisfies \( AA^{-}A = A \), use R to compute \( AA^{-}A \). Does this equal \( A \)?

```r
> A%*%gA%*%A
```

```
[,1] [,2] [,3]
[1,] 4  3  6
[2,] 2  6  2
```

(d) What is one solution to these equation (e.g., compute \( x = A^{-}y \)).

```r
> y <- matrix(c(6,4), nrow=2)
> gA%*%y
```

```
[,1]
[1,] 0.3870968
[2,] 0.3483871
[3,] 0.5677419
```

(e) Recall that a consistent system of equations satisfies \( AA^{-}y = y \). Is our system consistent?

```r
> A%*%gA%*%y
```

```
[,1] [,2]
[1,]  6  4
```

1
(f) Use R to compute \( I - A^{-1} \)
\[
\begin{align*}
> & \ I \leftarrow \text{matrix}(c(1, 0, 0, 0, 1, 0, 0, 0, 1), \text{nrow}=3) \\
> & \ I - \text{gA} \times \text{A}
\end{align*}
\]

\text{R} \text{ returns}
\[
[1,] \ 0.725806 \ -0.0967742 \ -0.4354839 \\
[2,] \ -0.0967742 \ 0.01290323 \ 0.05806452 \\
[3,] \ -0.4354839 \ 0.05806452 \ 0.26129032
\]

(g) Recall that for a consistent system, all solutions can be written as
\[ x = A^{-1}y + (I - A^{-1})c \], where \( c \) is any vector of constants. What is the family of solutions for this equation?

\[
\begin{align*}
& x_1 = 0.387 + 0.726c_1 - 0.097c_2 - 0.435c_3 \\
& x_2 = 0.348 - 0.097c_1 + 0.013c_2 + 0.058c_3 \\
& x_3 = 0.568 - 0.435c_1 + 0.058c_2 + 0.261c_3
\end{align*}
\]