Matrix Calculations in R
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R can be used to perform matrix multiplication and inversion. The syntax is a little odd, but straightforward. In the notes below, > indicates the R prompt, [1] the output from R

Defining Matrices

For starters, R is funny in that it works with column vectors. R starts with a list of elements and translates this into a matrix by filling up columns. The basic R command to define a matrix requires a list of elements (c(.,.,., .,)) and the number of rows nrow in the matrix. Consider the matrix

\[
C = \begin{pmatrix}
1 & 4 & 7 \\
2 & 5 & 8 \\
3 & 6 & 9
\end{pmatrix}
\]

To enter this matrix in R, we first have to write this as a single list, going down each column, i.e., c(1,2,3,4,5,6,7,8,9). To use R to set the variable C equal to the matrix C, we would use

> C <- matrix(c(1,2,3,4,5,6,7,8,9),nrow=3)

R uses the nrow command to set the dimension of the matrix. For example, if we entered

> C <- matrix(c(1,2,3,4,5,6,7,8,9),nrow=1)

This sets C equal to the matrix with a single row

\[
C = \begin{pmatrix}
123456789
\end{pmatrix}
\]

By typing C and hitting return, R displays the matrix C.

Conversely, you can instruct R to enter rows first by adding the command byrow=T, which enters the elements of the list as rows (the default is setting this option to false, entering this as columns). Thus entering

> D <- matrix(c(1,2,3,4,5,6,7,8,9),nrow=3,byrow=T)

returns the matrix

\[
D = \begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix}
\]

Individual elements can be extracted from a matrix C by using command C[i, j], which extracts the element in the ith row and jth column of C.
Matrix Transposition, \( t(C) \)

There are two ways to compute a transpose in \( C \). The simplest is to use the command \( t(C) \) to obtain the transpose of the matrix \( C \). One can also compute the transpose when entering a matrix by using the \( \text{byrow=T} \) command.

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**Example 1:** Using the \texttt{R} commands

\[
> E <- \text{matrix}(c(1,2,3,4,5,6), nrow=2) \\
> F <- \text{matrix}(c(1,2,3,4,5,6), nrow=3, byrow=T)
\]

Defines the matrices \( E \) and \( F \) as

\[
E = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}, \quad F = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}
\]

Note the \( E^T = F \). Likewise, one could also use

\[
> F <- t(E)
\]

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**Matrix Multiplication: \%\%\%**

To multiply two matrices, \texttt{R} uses the command \%\%\%. For example, using the matrices \( C \) and \( D \) above, the matrix product \( CD \) is computed in \texttt{R} by the command

\[
> C \%\%\% D
\]

\[
[,1] [,2] [,3] \\
[1,]  66  78  90 \\
[2,]  78  93 108 \\
[3,]  90 108 126
\]

Conversely, the matrix product \( DC \) is given by

\[
> D \%\%\% C
\]

\[
[,1] [,2] [,3] \\
[1,]  14  32  50 \\
[2,]  32  77 122 \\
[3,]  50 122 194
\]
Example 2: Consider the vector

\[ b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \]

Use R to compute the inner product \( b^T b \) and the outer product \( bb^T \).

```r
> b <- matrix(c(1,2,3),nrow=3)
> bt <- matrix(c(1,2,3),nrow=1)
> bt%*%b
 [,1] 
[1,] 14 
> b%*%bt
 [,1] [,2] [,3]
[1,] 1 2 3
[2,] 2 4 6
[3,] 3 6 9
```

The Inverse of a Matrix

The inverse of \( A \) is obtained using the solve command, with \( A^{-1} \) computed by `solve(A)`.

Example 3: Consider the following system of equations

\[
3x_1 + 4x_2 = 4 \\
x_1 + 6x_2 = 2
\]

In matrix form, this becomes \( Ax = y \) where

\[
A = \begin{pmatrix} 3 & 4 \\ 1 & 6 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad y = \begin{pmatrix} 4 \\ 2 \end{pmatrix}
\]

Hence, \( x = A^{-1}y \), or in R

```r
> A <- matrix(c(3,1,4,6),nrow=2)
> y <- matrix(c(4,2),nrow=2)
> x<- solve(A)%*%y
> x
```
Matrix Calculations in R

R returns

\[
\begin{bmatrix}
[,1] \\
[1,]\quad 1.1428571 \\
[2,]\quad 0.1428571 \\
\end{bmatrix}
\]

We can check this by looking at the first equation, \(3x_1 + 4x_2 = 4\)

\[
\begin{bmatrix}
3\times[1,1]+4\times[2,1] \\
\end{bmatrix}
\]

R returns

\[
\begin{bmatrix}
1 \\
\end{bmatrix}
\]

Eigenvalues, vectors of a Matrix

The command \texttt{eigen(X)} returns the eigenvalues and vectors of for the square matrix \(X\).

Example 4: Suppose we are still in R with \(A\) as in Example 3.

\[
\begin{bmatrix}
[1,] \\
[1,]\quad -0.8246211 \\
[2,]\quad -0.8246211 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
[2,] \\
[1,]\quad -0.9701425 \\
[2,]\quad 0.2425356 \\
\end{bmatrix}
\]
