

Power Calculations in R

The notes supplement both the "Introduction to R" notes and the notes of Computing power. To remind the reader how to compute p and critical values for the distributions useful in power calculations (normal, t , χ^2 , F), here are a number of examples. The reader should also work through these examples in R.

The `>` indicates the R prompt, `[1]` the output from R

Normals

- Compute the 99% **quantile** (the value correspond to a the 99% probability) for a unit normal.

```
> qnorm(0.99)
[1] 2.326348
```

We used the notation $z_{(\alpha)}$ in the power notes (eq., Equations A5.1a-c) as the solution to $\Pr(U \leq z_{(\alpha)}) = \alpha$ where U is the unit normal. This is just the α quantile. For example, to compute $z_{(0.01)}$,

```
> qnorm(0.01)
[1] -2.326348
```

- Compute the probability that a unit normal is less than or equal to -0.35.

```
> pnorm(-0.35)
[1] 0.3631693
```

Note that **q** indicates computing a quantile (given a probability, R returns a number), while **p** computes a probability (given a value, it returns the probability of being less than or equal to that value)

Student's t

- Compute the 95% quantile for a t distribution with 19 degrees of freedom.

```
> qt(0.95,19)
[1] 1.729133
```

- Compute the probability that a t random variable with 6 degrees of freedom is greater than 1.1

```
> 1-pt(1.1,6)
[1] 0.1567481
```

χ^2 distribution

- Compute the 90% quantile for a (central) χ^2 distribution for 15 degrees of freedom

```
> qchisq(0.9,15)
[1] 22.30713
```

- Compute probability that a (central) χ^2 distribution with 13 degrees of freedom is less than or equal to 21.

```
> pchisq(21,13)
[1] 0.9270714
```

- Compute probability that a χ^2 distribution with 13 degrees of freedom and noncentrality parameter 5.4 is less than or equal to 21.

```
> pchisq(21,13,5.4)
[1] 0.6837649
```

- Compute the 50% quantile for a χ^2 distribution with 7 degrees of freedom and noncentrality parameter 3.

```
> qchisq(0.5,7,3)
[1] 9.180148
```

F distribution

- Compute the value C for an F distribution with 3 and 16 degrees of freedom such that $\Pr(F_{3,16} > C) = 0.05$ (This value is used in Example 3 in the power notes)

```
> qf(1-0.05,3,16)
[1] 3.238872
```

- Compute the probability that a noncentral F with 3 and 16 degrees of freedom and noncentrality parameter 5 exceeds 3.24 (from Example 3)

```
> 1-pf(3.24,3,16,5)
[1] 0.3533815
```

Plotting Power Curves

Using some of the graphics commands introduced in the Introduction to **R** Notes, we can compute various power curves. Be sure and do the graphics examples from the Introduction Notes first. Some other useful coding in **R** are as follows:

- `sqrt(x)` – returns \sqrt{x}
- Defining functions. Suppose we wish to define $g(x, y, z) = \log(x + y)/z$. In **R** we do this by using the syntax

```
> g <- function(x,y,z) log(x+y)/z
```

For example, consider Equation A5.2b in the power notes, the α -level critical value when the mean is μ_0 , the variance is $\sigma_0^2 = f_0^2/n$, $T_c(\alpha) = \mu_0 + \sigma_0 z_{(1-\alpha)}$. In **R** we can write this function as

```
Talpha <- function(mu0,f0,n,alpha) mu0 +sqrt(f0/n)*qnorm(1-alpha)
```

To compute the critical value for $\alpha = 0.01$, $\mu_0 = 12$, $f_0 = 3$, and $n = 25$,

```
> Talpha(12,3,25,0.01)
[1] 12.80587
```

Likewise, if the true mean is μ_1 and the true variance is $\sigma_1^2 = f_1^2/n$, the power for an α -level test is given from Equation A5.3, $\Pr(U > [T_c(\alpha) - \mu]1/\sigma_1)$, and we can use the previous function write this in R as the function

```
Palpha <- function(mu0,mu1,f0,f1,n,alpha)
1-pnorm( (Talpha(mu0,f0,n,alpha)-mu1)/sqrt(f1^2/n))
```

We can this use to plot various power functions. For example, how does power change as we change the true mean? Suppose $\mu_0 = 5$, $f_0 = 3$, $f_1 = 2$, $n = 50$ and $\alpha = 0.1$. To plot the power for μ_1 ranging from 0 to 15, in R use the above function and type

```
> curve(Palpha(5,x,3,2,50,0.1), 0,15)
```

Note that the syntax for the cruve function is to use x as the variable to to change, so that `cruve(f(x),a,b)` plots how $f(x)$ changes from $x=a$ to $x=b$.

To see how power changes as we vary n , assume that $\mu_1 = 6$, and vary n from 20 to 300

```
> curve(Palpha(5,6,3,2,x,0.1), 20,300)
```

Iterative Power Calculations

As a final example of computing power, consider Example 3 in the Power notes, on computing the power for an ANOVA. In this example there are $N = 4$ treatments that account for $1/3$ of all variance. As shown in the example, with n replicates per treatment, this implies the noncentrality parameter of the F distribution is $= (N - 1)n(1/3) = n$. We increase power by increasing the number of replciates per treatment n , but this also requires us to recompute the critical value (which also changes with n). As we increase n , the F degrees of freedom become 3 and $4(n - 1)$. The 95% critical value for any n can be computed in R by defining the function

```
Fcrit <- function (n) qf(0.95,3,4*(n-1))
```

For example,

```
> Fcrit(5)
[1] 3.238872      (the value in Example for n=5)
> Fcrit(15)
[1] 2.769431      (the value in Example for n=15)
```

To compute the power for various values of n , use the above function **Fcrit** and define the function **Fpower** (or whatever you wish to call it) by

```
> Fpower <- function(n) 1-pf(Fcrit(n),3,4*(n-1),n)
> Fpower(5)
[1] 0.3535594      (the power for n=5)
> Fpower(15)
[1] 0.8957212      (the power for n =15)
> Fpower(16)
[1] 0.9167217      (the power for n=16)
```