Statistical Genetics
Agronomy 615
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EXERCISES FOR CHAPTER 6

Exercise 6.1. Give as many reasons as you can why F is a useful parameter in population and quantitative genetics.

Exercise 6.2. Suppose that from a large random-mating population with a known frequency 0.40 for heterozygosity at a particular locus, one randomly chooses the progenitors for a large number of lines.

a. Assume that for each line the system of mating practiced was self-fertilization. What is the probability of heterozygosity at that locus in any random line after five generations of selfing? What is the probability of homozygosity in any line?

b. Repeat (a) above but assume that the mating system of double first cousins was used.

c. For each of the mating systems, self-fertilization and double first cousins, what is the proportion of those independent loci which were originally segregating in the random-mating population but are now nonsegregating after five generations of inbreeding?

Exercise 6.3.

a. Derive the recurrence relation of the inbreeding coefficient for the mating of quadruple second cousins in a manner similar to that for the mating of double first cousins. The desired recurrence relation is

$$F_t = \frac{1}{16} + \frac{1}{2}F_{t-1} + \frac{1}{4}F_{t-2} + \frac{1}{8}F_{t-3} + \frac{1}{16}F_{t-4}$$

The pedigree is shown in Figure 6.5, p. 6.29, and will not be reproduced here. It is helpful to note that the following relationships exist:

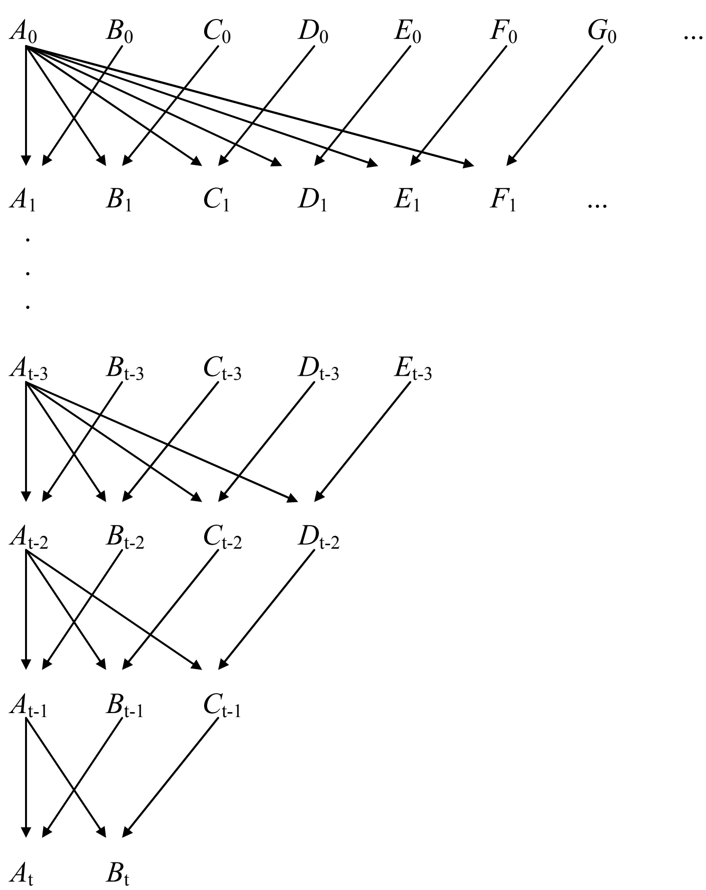
Full sibs:	A and E,	B and F
	C and G,	D and H
Double first cousins:	A and C,	A and G
	E and C,	E and G
	B and D,	B and H
	F and D,	F and H
Quadruple second cousins:	A and B,	A and F
	E and B,	E and F
	C and D,	C and H
	G and D,	G and H
	A and D,	A and H
	E and D,	E and H
	B and C,	B and G
	F and C,	F and G

They may be summarized as follows:

	A	B	C	D	E	F	G	H
A		QSC	DFC	QSC	FS	QSC	DFC	QSC
B			QSC	DFC	QSC	FS	QSC	DFC
C				QSC	DFC	QSC	FS	QSC
D					QSC	DFC	QSC	FS
E						QSC	DFC	QSC
F							QSC	DFC
G								QSC
H								

b. For such a closed mating line or group described above, derive the general expression for the mean coefficient of coancestry, $\bar{\theta}_t$, between members of the group.

Exercise 6.4. One kind of half-sib mating is represented by the following pedigree. Individuals in generation 0 are all noninbred and unrelated.



Derive the recurrent relation for F_t, and give the recurrent series for generations 0 to 7.

Exercise 6.5. What kind of backcrossing system and conditions would give a constant inbreeding coefficient of 1/2 every successive generation of backcrossing? Show that bisexual identical twins have a coefficient of coancestry of 1/2.

Exercise 6.6. Many authors state that the effective population number for separate sexes is (e.g., J.F. Crow and M. Kimura, 1970, An Introduction to Population Genetics Theory, Harper & Row, p. 103)

$$N_e = \frac{1}{\frac{1}{4N_m} + \frac{1}{4N_f}}$$

However, for full-sib mating $N_m = N_f = 1$, this formula becomes $N_e = 2$. Is this number really the effective population number which approximately equates the rate of inbreeding to that of the ideal population of a monoecious species with random self-fertilization. Clarify the source of confusion.

Exercise 6.7.

a. Suppose that an animal breeder wanted to inbreed some finite population to a certain level, and then to expand that population to an infinitely large size to maintain the same level of inbreeding. He decided to inbreed by the double-first-cousin system. He randomly chose four individuals (N = 4) from a large noninbred population, generation 0, and inbred until he had individuals in generation 4 with $F_4 = 3/16$. He continued to breed the same parental individuals in geneneration 3, mating them in the same manner as that in the double-first-cousin system, until generation 4 was infinitely large. Then he randomly mated the infinitely large population thereafter, labeling the subsequent generations as 5, … , ∞. What is the inbreeding coefficient in generation 5? In generation ∞?

b. Alternatively, let us suppose that the experimenter had randomly mated the bisexual parental individuals in generation 3 to produce an infinitely large generation 4, permitting random self-fertilization. How does the inbreeding coefficient in generation 4 compare with that in generation 4 in part (a) above. What about in generation 5? What about in generation ∞? Are there any differences?

Exercise 6.8. Suppose that an experimenter, working with a laboratory animal, found a single recessive gene, *a*, in some heterogeneous stock. He desired to compare the expression of the three genotypes, *AA*, *Aa*, and *aa*, in a single inbred line. The alternative procedures which he considered were full-sib mating or backcrossing to a pre-existing fully inbred *AA* line. Because the allele *a* is completely recessive to *A*, carriers must be identified by observing the offspring from an appropriate mating. Consider only the case of a fully viable, fertile homozygous recessive. Assume no particular restrictions in the biology of the animal. Develop the most efficient breeding procedure as you can for each of the two procedures—both in terms of number of generations and number of individuals bred. Diagram your pedigrees for each and give or calculate the inbreeding coefficient for a few generations. Which system would you prefer? Why?

Exercise 6.9.

a. Discuss why N_{t-1} is the number that influences F_t for a monoecious species with random self-fertilization.
b. Discuss why N_{t-2} is the number that influences F_t for a monoecious species with avoidance of selfing, a dioecious species with polygamy, and that with monogamy.

Exercise 6.10. Give the probability argument in developing the recurrence relation of F for each of the four kinds of finite populations with nonregular mating system. Assume a constant population size from generation to generation.

a. Monoecious population with random selfing [equation (6.133)].
b. Monoecious population with avoidance of self-fertilization [equation (6.149)].
c. Dioecious population with random mating with random mating (polygamy) [equation (6.174)].
d. Dioecious population with random mating of mates (monogamy) [equation (6.190)].

Exercise 6.11. Discuss the effects of bottlenecks or restrictions in population size in one or two generations upon the effective population number.

Exercise 6.12. A maize or corn (*Zea maize*) breeder is interested in maintaining a germplasm stock. Maize is a monoecious species with self-fertilization possible.

a. Suppose that a breeder establishes 10 plants in an isolation block, allows the 10 plants to random mate with random self-fertilization permitted. The breeder harvests one ear from each plant. Then the breeder shells (or removes all kernels from each ear) and bulks the kernels from all 10 ears together. All ears are assumed to have the same number of kernels (about 500). Then a random sample of sufficient size from the bulk is planted to establish 10 breeding plants for the next generation. Give the formula for the effective population number for such a system. Specify what each symbol is numerically, and substitute them into the appropriate formula for the effective population number to calculate its numerical value. What is the rate of inbreeding? (The number 10 is chosen to be arbitrarily small for ease in computation).

b. Suppose that the breeder has the same situation as in (a) above, except that instead of bulking the seed from all 10 ears, the breeder harvests only two or three kernels from each ear, plants them, and reduces that number to one plant per ear, giving 10 plants in total which is the same number as in the previous generation. Give the formula for the effective population number for such a system. Specify what each symbol is numerically, and substitute them into the appropriate formula for the effective population number to calculate its numerical value. What is the rate of inbreeding?

c. Suppose that the breeder has the same situation as in (a) above, except a random sample of sufficient size from the bulk is planted to establish 20 plants for the next generation -- all of which function with equal chance as male parents. However, only one ear from 10 randomly chosen plants out of the 20 is harvested. Kernels from all 10 ears are bulked and used to establish the 20 plants mentioned above. Does this system alter the effective population number from that in (a) above? Give the formula for the effective population number for such a system. Specify what each symbol is numerically, and substitute them into the formula for the effective population number to calculate its numerical value. Note that the number of breeding males, N_m, in every generation is 20, and that for the breeding females, N_f, is 10. Explain the rationale behind the arguments for what you do. Hint: A close study of Section 6.4.3.2.1 should give the necessary clues. An appropriate diagram similar to (6.208) was helpful to me.

Exercise 6.13. Discuss the similarities in the definitions between the coefficient of inbreeding of an individual and that of a group, between the coefficient of coancestry between two individuals and that between two groups, and the coefficient of coancestry of an individual with itself and that of a group with itself.