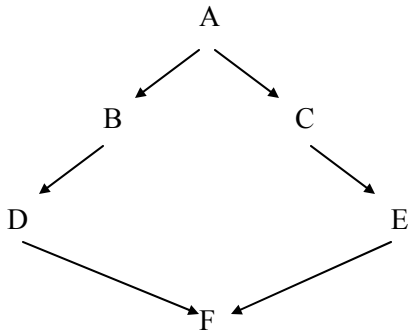


EXERCISES FOR CHAPTER 4

**Exercise 4.1.** What is the coefficient of coancestry between grandfather and grandson in a large random-mating population?

**Exercise 4.2.** What is the coefficient of coancestry between the offspring of a pair of monozygotic twins married to unrelated spouses?

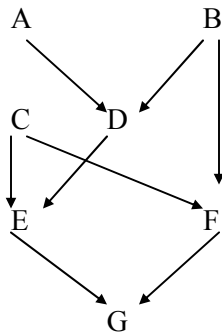
**Exercise 4.3.** Suppose that one is given the following pedigree where capital letters designate unisexual individuals, i.e., the parents of *B* are *A* and *another* unspecified individual. All relationships other than those given by the diagram are assumed to be zero.



- By use of a table, calculate the coefficients of coancestry between all individuals in the pedigree.
- What is the coefficient of inbreeding of individual *F* (or the last individual in the pedigree) as given in the table?
- Calculate the coefficient of inbreeding of individual *F* (or the last individual in the pedigree) by expanding backwards through the use of rules 1 and 2.
- Calculate the coefficient of inbreeding of individual *F* (or the last individual in the pedigree) by using equation (4.60).

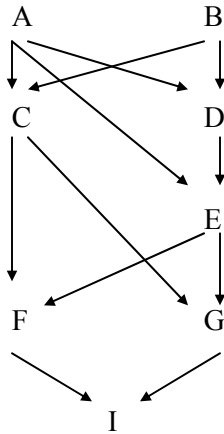
**Exercise 4.4.** Repeat Exercise 4.3 except assume that  $F_A = 1/2$ .

**Exercise 4.5.** Develop a table of coefficients of coancestries for the individuals in the following pedigree where we assume  $F_A = F_B = \theta_{AB} = \theta_{AC} = \theta_{BC} = 0$ ,  $F_C = \frac{1}{5}$ . (Note:  $F_C = \frac{1}{5}$ )

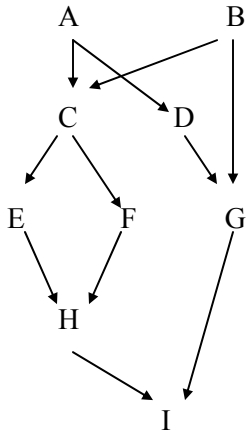


What is the inbreeding coefficient of individual *G*? If the next mating could be either  $C \times G$  or  $D \times G$  and inbreeding is to be minimized, which of the two matings would you recommend?

**Exercise 4.6.** For the pedigree given below, do the same as in Exercise 4.3.

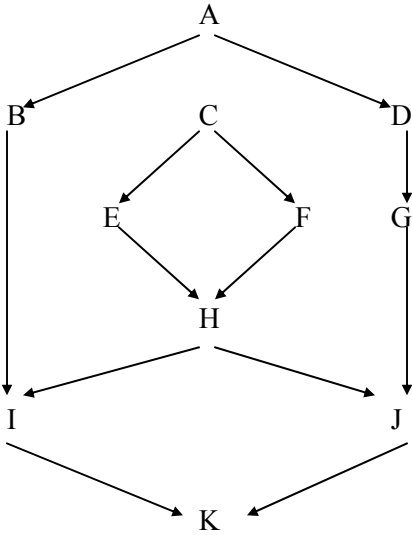


**Exercise 4.7.** Develop a table of coefficients of coancestries for the following pedigree of individuals where we assume  $F_A = F_B = \theta_{AB} = \theta_{BD} = 0$ .



What is the coefficient of inbreeding of individual  $I$ ? Also, calculate the coefficient of inbreeding of individual  $I$  by summing the contributions due to the chains of coancestries.

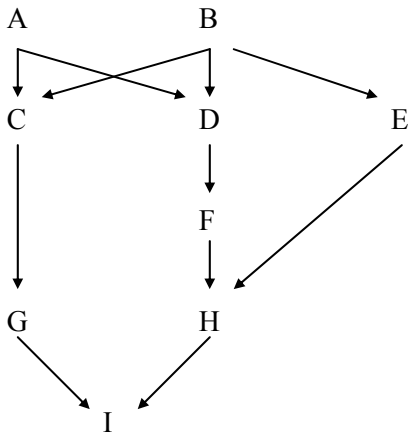
**Exercise 4.8.** Development a table of coefficients of coancestry for the following pedigree of individuals where we assume  $F_A, F_C$ , and  $\theta_{AC} = 0$ .



- What is the coefficient of inbreeding of individual K?
- Also calculate the coefficient of inbreeding of individual K by summing the contributions due to the chains of coancestries.

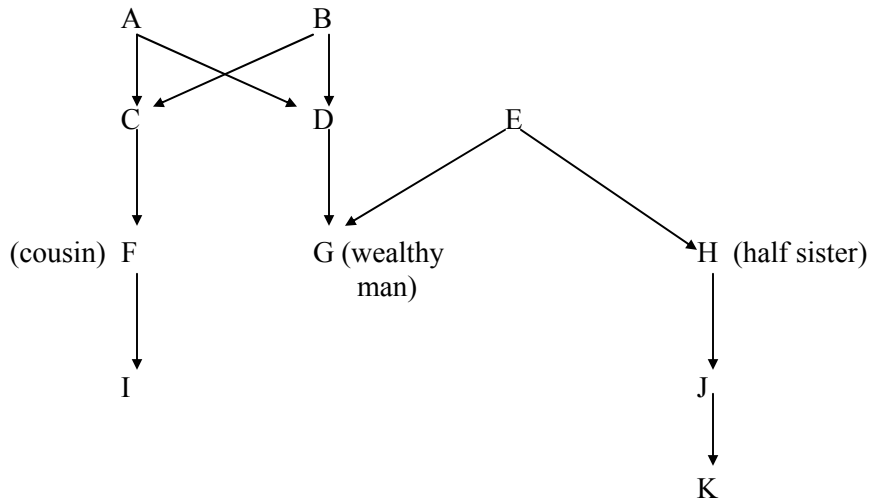
**Exercise 4.9.**

- Calculate the coefficients of coancestries for the following pedigree, assuming  $\theta_{AB} = F_A = F_B = 0$ .

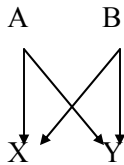


- What is the inbreeding coefficient of I?

**Exercise 4.10.** A wealthy man died and left all of his money to his nearest relative. Of the two candidates, one was the son of his cousin, and the other was the grandson of his half sister. Who received the money?



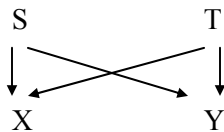
**Exercise 4.11.** The parents,  $A$  and  $B$ , of the full sibs,  $X$  and  $Y$ , are unrelated, but are inbred,  $F_A = 1/2$ ,  $F_B = 3/4$ .



- What is the probability that a random gene of  $X$  is identical by descent to a random gene of  $Y$ ?
- What is the coefficient of inbreeding of an offspring from the mating of  $A$  with  $X$ ?
- What is the probability that the *genotype* (for a single locus) of  $X$  is identical by descent to that of  $Y$ ?

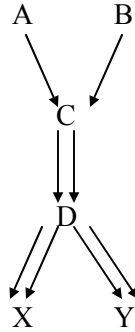
(Hint: suppose that  $A$  has genes  $a$  and  $b$  and that  $B$  has genes  $c$  and  $d$ ).

**Exercise 4.12.** In a full-sib pedigree where  $\theta_{ST} = 0$ ,



- Give a direct argument or proof why  $\gamma_{\ddot{X}Y}$  equals zero (do not use the rules).
- Give a direct argument or proof why  $\gamma_{XXY}$  equals  $1/8$  (do not use the rules). Contrast the answer in this part with that in part (a). Discuss.

**Exercise 4.13.** Consider the following pedigree in a self-fertilizing plant species



Assuming,  $\delta_{\ddot{A}\ddot{B}} = \Delta_{\ddot{A}\ddot{B}} = \Delta_{\ddot{A}+\ddot{B}} = \gamma_{\ddot{A}\ddot{B}} = \gamma_{\ddot{A}\ddot{B}} = F_{\ddot{A}} = F_{\ddot{B}} = \theta_{AB} = 0$ , calculate the condensed coefficients of identity for individuals  $X$  and  $Y$ . For every rule used, give its number above the term to which you applied the rule. Do the same for the assumptions used. The rules and their numbers are listed in Table 4.9, pp. 4.104 to 4.105. Also, do not forget equation (4.86), which should have been given a rule number and listed in Table 4.9. You may use the tabular procedure to calculate the coefficients of coancestry and inbreeding, if you like. After calculating the condensed coefficients of identity, check to see that they add to one.

**Exercise 4.14.** Suppose the mean coefficient of inbreeding of individuals in some population was equal to 0.29 as calculated relative to some base population which had been assumed to have no inbreeding. However, later evidence showed that the base population had a mean coefficient of inbreeding equal to 0.11 relative to some earlier population. What is the actual amount of inbreeding in the present population relative to this earlier population?

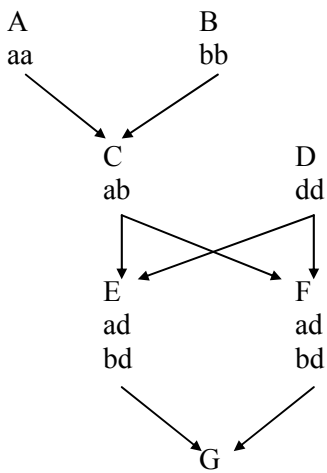
**Exercise 4.15.**

a. Suppose a maize two-way hybrid (a cross between two unrelated, fully inbred lines) is allowed to undergo open pollination (i.e., random mating) for one generation. What is the inbreeding coefficient of a random offspring? Deduce from elementary consideration of the genes involved. Alternatively, consider two simple possible diagrams that would represent the situation (consider a single individual for each parent in both diagrams), and calculate the inbreeding coefficient of the offspring individual(s) for each. Determine that all three procedures give the same answer. What property permits simple diagrams to represent this random-mating situation? (In answering this question, consider genes that are truly identical by descent.)

b. Consider the open-pollinated progeny of a three-way hybrid, again using fully inbred lines. Deduce the inbreeding coefficient from elementary consideration of the genes involved. Alternatively, consider a simple diagram, and calculate the inbreeding coefficient of the offspring individuals.

**Exercise 4.16**

a. Let us reconsider Exercise 4.15.b, i.e., the random-mated or open-pollinated progeny of a three-way hybrid, using fully inbred lines. The coefficient of inbreeding was  $F = 3/8$ . This was deduced from elementary means as follows:



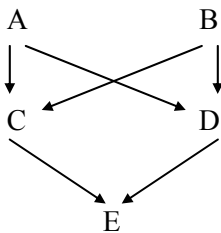
	a 1/4	d 1/4	b 1/4	d 1/4
a	aa*	ad	ab	ad
d	ad	dd*	bd	dd*
b	ab	bd	bb*	bd
d	ad	dd*	bd	dd*

\*identical by descent

$$F = 6/16 = 3/8$$

This was also obtained by considering the individual  $G$  obtained from the cross between  $E$  and  $F$ . Develop a table of coancestries to show that the inbreeding of  $G$  equals  $3/8$ . Why can you represent the amount of inbreeding of the offspring from random mating among equal proportions of  $E$  and  $F$  individuals by the inbreeding of the individual  $G$ ? Does the inbreeding coefficient of  $G$  change if  $E$  is represented by only one individual and  $F$  by only one individual in contrast to random mating among a large number of individuals of the three-way cross? Is anything different with respect to  $F$  between the two crossing situations just described? (Hint: Note that the inbreeding coefficient in the upper left and the lower right  $2 \times 2$  subtables have 2 of 4 cells identical by descent, and the other  $2 \times 2$  subtables have only 1 of 4 cells identical by descent. Consider this effect upon the variance of  $F$ , the inbreeding coefficient.)

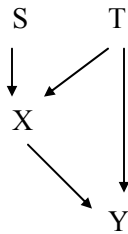
b. Next, let us return to the maize two-way hybrid of Exercise 4.15.a. In this case, suppose the cross was between two  $S_1$  lines ( $F_A = F_B = 1/2$ ), using a large number ( $>100$ ) of individuals in each line. Then open-pollinate (i.e., random mate) the cross for one generation. The pedigree is



What is the inbreeding coefficient of the random offspring  $E$ ? Deduce from elementary considerations of the genes involved. Develop a table of coefficients of coancestries, assuming  $A, B, C,$  and  $D$  are individuals. Are the results the same? Why or why not?

**Exercise 4.17.** What is meant by a four-gene probability? Distinguish between a four-gene probability for which only three genes are identical by descent and a three-gene probability for which all three genes are identical by descent.

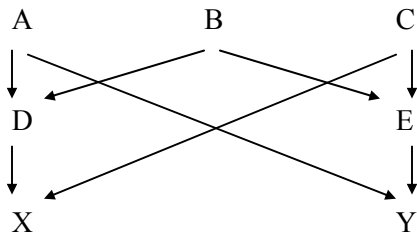
**Exercise 4.18.** Consider the following pedigree



Assuming  $\delta_{\ddot{S}\ddot{T}} = \Delta_{\ddot{S},\ddot{T}} = \Delta_{\ddot{S}+\ddot{T}} = \gamma_{\ddot{S}\ddot{T}} = \gamma_{\ddot{S}\ddot{T}} = F_{\ddot{S}} = F_{\ddot{T}} = \theta_{\ddot{S}\ddot{T}} = 0$  calculate the condensed coefficients of identity for individuals  $X$  and  $Y$ . For every rule used, give its number above the term to which you applied the rule, as I did in the examples in Section 4.11. The rules and their numbers are listed in Table 4.9, pp. 4.104 to 4.105. Do the same for the assumptions used. After calculating the condensed coefficients of identity, check to see that they add to one. (Note: This pedigree is the mating between the younger parent and its offspring (see Chapter 6, Section 6.3.4.2). The condensed coefficients of identity calculated herein may well have some practical application in the estimation of genetic variance in the study of genetic variation.)

Answers (intermediate):  $\delta_{\ddot{X}\ddot{Y}} = 0, \Delta_{\ddot{X},\ddot{Y}} = 0, \Delta_{\ddot{X}+\ddot{Y}} = \frac{1}{8}, \gamma_{\ddot{X}\ddot{Y}} = 0, \gamma_{\ddot{X}\ddot{Y}} = \frac{1}{8}, F_{\ddot{X}} = 0, F_{\ddot{Y}} = \frac{1}{4}, \theta_{\ddot{X}\ddot{Y}} = \frac{3}{8}$

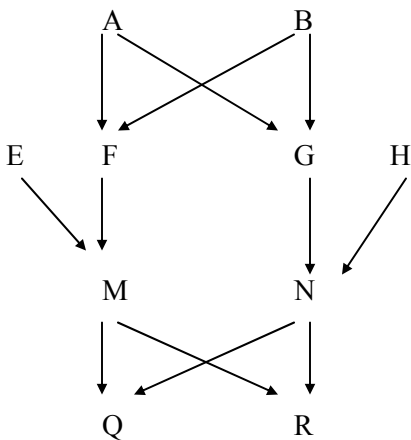
**Exercise 4.19.** Consider the following pedigree of two different three-way crosses involving the same parents



We assume  $\delta_{\ddot{A}\ddot{B}} = \Delta_{\ddot{A},\ddot{B}} = \Delta_{\ddot{A}+\ddot{B}} = \gamma_{\ddot{A}\ddot{B}} = \gamma_{\ddot{A}\ddot{B}} = F_{\ddot{A}} = F_{\ddot{B}} = \theta_{\ddot{A}\ddot{B}} = 0$ , similarly for  $A$  and  $C$ , and similarly for  $B$  and  $C$ . Calculate the condensed coefficients of identity for individuals  $X$  and  $Y$ . For every rule used, give its number above the term to which you applied the rule, as I did in the examples in Section 4.11. The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Do the same for the assumptions used. After calculating the condensed coefficients of identity, check to see that they add to one. (Note: The condensed coefficients of identity calculated herein may well have some practical application in the estimation of genetic variance in the study of genetic variation.)

**NOTE:** All of the Exercises 4.20 to 4.27 involve subsets of the individuals in the complex pedigree given by Albert Jacquard in his book entitled, *The Genetic Structure of Populations*, Springer Verlag, 1974, p. 113. The exercises were developed sequentially in successive years. The long-range objective was to calculate the condensed coefficients of identity by descent,  $\Delta_1$  to  $\Delta_9$ , for the last two individuals in the pedigree designated  $U$  and  $V$  by Nyquist (designated  $A$  and  $B$  by Jacquard) and compare their values to those given by Jacquard on p. 114. Hopefully the same values would be obtained, barring possible errors. How much more work is required to accomplish that objective was not pursued. Please note in the footnote (\*) to equation (4.82) that the condensed coefficients of identity by descent in my notes have been labeled differently from those by Jacquard.

**Exercise 4.20.** Albert Jacquard, 1974, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree of which the following is only a part. The individuals in the original pedigree in Jacquard were given letters sequentially according to the order in the English alphabet, starting with  $A$  and  $B$  in the first generation. The letter designations are retained in the pedigree presented here.



Assuming all two-, three-, and four-gene states of identity, involving one, two, three, and four individuals,  $A$ ,  $B$ ,  $E$  and  $H$ , equal zero, calculate the condensed coefficients of identity for individuals  $Q$  and  $R$ . For every rule used, give its number above the term to which you applied the rule. Do the same for the assumptions by using the letter  $A$ . The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Also, do not forget equation (4.86), which should have been given a rule number and listed in Table 4.9.

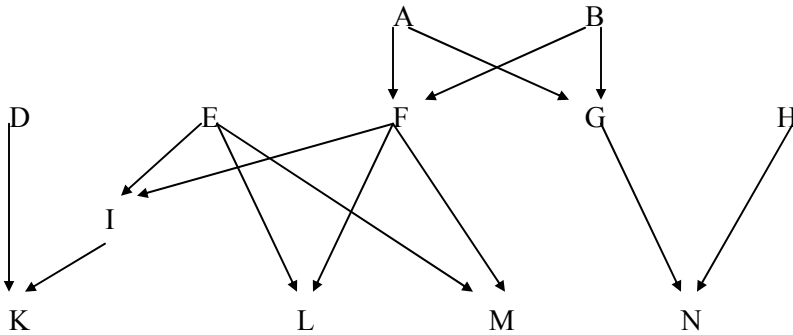
For thoroughness, one would continue to apply the proper rule until you have all probabilities in terms of only individuals  $A$ ,  $B$ ,  $E$ , and  $H$  with no individual letters repeated. One would then indicate the setting of all such probabilities equal to zero by using the letter "A". We assume that all two, three, and four-gene states of identity (except those of no pairs identical) involving any of the individuals  $A$ ,  $B$ ,  $E$ , and  $H$  are zero (note  $F$  and  $G$  are not included). The only relations that exist are those specifically given by the pedigree. Also assume  $F_A = F_B = F_E = F_H = 0$ .

Although the continued expansion implied above is what one would do for completeness sake, a lot of work can be avoided by doing the following. For example, a probability such as  $\delta_{\overline{MN}}$  may be recognized as being equal to zero, because of the two genes in  $M$  one comes from  $E$  and the other comes from  $F$ . Since  $E$  and  $F$  are unrelated,  $\delta_{\overline{MN}}$  must be equal to zero because it is impossible for *all* four genes to be identical by descent. A similar argument could be used for the two genes in  $N$ . Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. I highly encourage you to follow this procedure.

Any probability which has already been worked out in my notes, you may simply substitute the numerical value and cite the particular page number, equation, and/or example number.

After calculating the condensed coefficients of identity, check to see that they add to one.

**Exercise 4.21.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree of which the following is only a part.



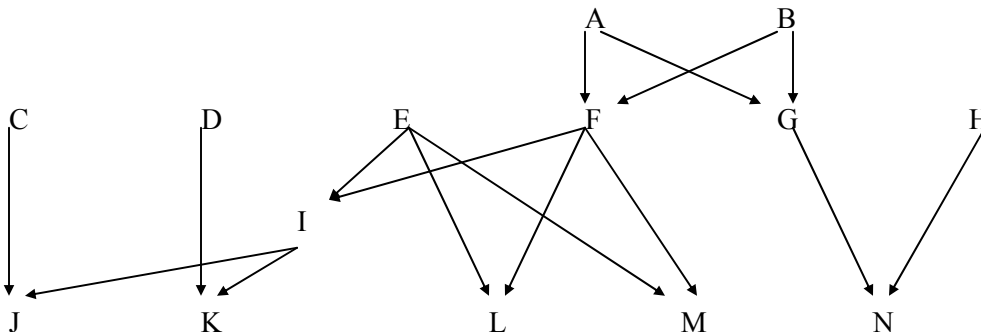
Calculate  $\gamma_{KLM}$  (all three genes identical by descent) and  $\delta_{KLMN}$  (all four genes identical by descent). For every rule used, give its number above the term to which you apply the rule. The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Also do not forget equation (4.86) which should have been given a rule number and listed in Table 4.9.

For thoroughness, one would continue to apply the proper rule until you have all probabilities in terms of only individuals  $A, B, D, E,$  and  $H$  with no individual letters repeated. One would then indicate the setting of all such probabilities equal to zero by using the letter "A". We assume that all two, three, and four-gene states of identity (except those of no pairs identical) involving any of the individuals  $A, B, D, E,$  and  $H$  are zero (note  $F$  and  $G$  are not included). The only relations that exist are those specifically given by the pedigree. Also assume  $F_A = F_B = F_D = F_E = F_H = 0$ .

Although the continued expansion implied above is what one would do for completeness sake, a lot of work can be avoided by doing the following. For example, a probability such as  $\gamma_{DMN}$  may be recognized as being equal to zero, because  $D$  is unrelated to the ancestor,  $E, A, B,$  and  $H$ , which contribute genes to  $M$  and  $N$ . Therefore, since it is impossible for the three random genes, one from each of the individuals  $D, M,$  and  $N$ , to be all identical,  $\gamma_{DMN}$  may be declared to be equal to zero. Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. I highly encourage you to follow this procedure.

Any probability which has already been worked out in my notes, you may simply substitute the numerical value and cite the particular page number, equation, and/or example number.

**Exercise 4.22.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree of which the following is only a part.



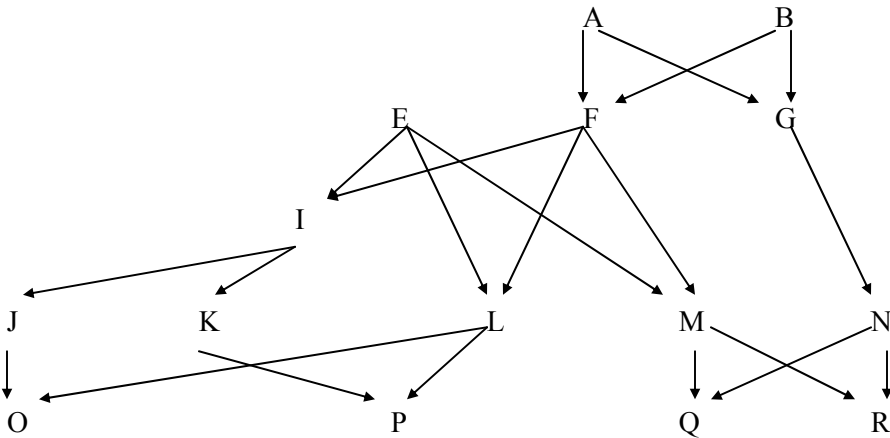
Calculate  $\gamma_{KLN}, \gamma_{KMN},$  and  $\gamma_{LMN}$  (all three genes identical by descent) and  $\delta_{JKLM}$  and  $\delta_{JKLN}$  (all four genes identical by descent). For every rule used, give its number above the term to which you apply the rule. The rules and their numbers are listed in Table 4.9, p. 4.101. Also do not forget equation (4.86) which should have been given a rule number and listed in Table 4.9.

For thoroughness, one would continue to apply the proper rule until you have all probabilities in terms of only individuals  $A, B, D, E,$  and  $H$  with no individual letters repeated. One would then indicate the setting of all such probabilities equal to zero by using the letter "A". We assume that all two, three, and four-gene states of identity (except those of no pairs identical) involving any of the individuals  $A, B, C, D, E,$  and  $H$  are zero (note  $F$  and  $G$  are not included). The only relations that exist are those specifically given by the pedigree. Also assume  $F_A = F_B = F_C = F_D = F_E = F_H = 0$ .

Although the continued expansion implied above is what one would do for completeness sake, a lot of work can be avoided by doing the following. For example, a probability such as  $\gamma_{DMN}$  may be recognized as being equal to zero, because  $D$  is unrelated to the ancestors of  $M$  and  $N$  ( $E, A, B,$  and  $H$ ), which contribute genes to  $M$  and  $N$ . Therefore, since it is impossible for the three random genes, one from each of the individuals  $D, M,$  and  $N,$  to be all identical,  $\gamma_{DMN}$  may be declared to be equal to zero. Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. I highly encourage you to follow this procedure.

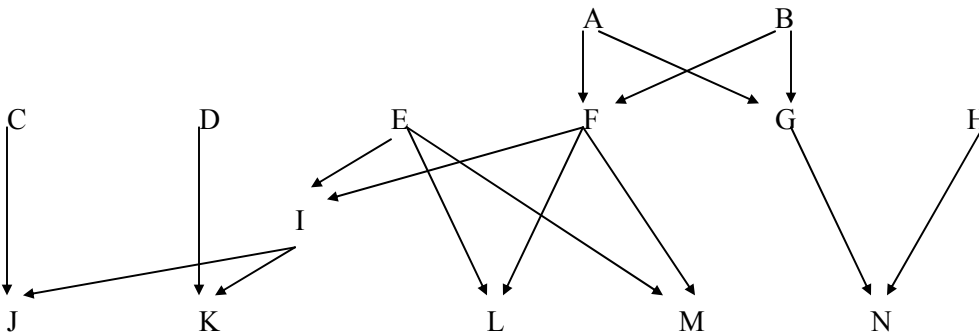
Any probability which has already been worked out in my notes, you may simply substitute the numerical value and cite the particular page number, equation, and/or example number.

**Exercise 4.23.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree.



Develop a table of coefficients of coancestries where we assume that  $A, B,$  and  $E$  are unrelated, noninbred individuals. What is the coefficient of inbreeding of  $O, P, Q,$  and  $R$ ?

**Exercise 4.24.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree of which the following is only a part.



For the generation represented by individuals  $J, K, L, M,$  and  $N,$  calculate  $\gamma_{JKL}, \gamma_{JKM}, \gamma_{JKN}, \gamma_{JLM}, \gamma_{JLN},$  and  $\gamma_{JMN}$  (all three genes identical by descent) and  $\delta_{JKMN}$  and  $\delta_{JLMN}$  (all four genes identical by descent). (Note that  $\gamma_{JKL} = \gamma_{JKM} = \gamma_{JK(EF)},$  so you need to solve for only one.)

The following probabilities are given for a somewhat earlier generation ( $I, L, M,$  and  $N$ ). The ones given are the four possible all-three-genes identical-by-descent in three individuals and the one all-four-genes identical-by-descent in four individuals. The probabilities are:

$$\gamma_{ILM} = 1/16, \quad \gamma_{ILN} = 1/64, \quad \gamma_{IMN} = 1/64, \quad \gamma_{LMN} = 1/64$$

$$\delta_{ILMN} = 1/256$$

The above quantities can be used very effectively to obtain  $\gamma_{JLM}, \gamma_{JLN},$  and  $\gamma_{JMN}$  by expanding the descendant  $J$  of individual  $I$ .

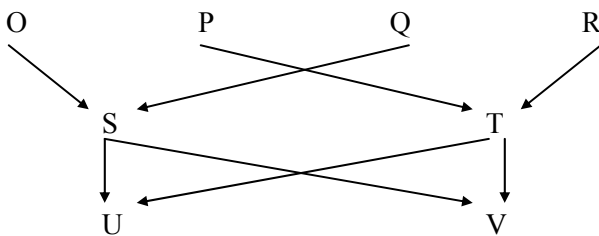
For the rules used, give its number above the term to which you apply the rule. The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Also do not forget equation (4.86) which should have been given a rule number and listed in Table 4.9.

For thoroughness, one would continue to apply the proper rule until you have all probabilities in terms of only individuals  $A, B, C, D, E,$  and  $H$  with no individual letters repeated. One would then indicate the setting of all such probabilities equal to zero by using the letter "A". We assume that all two, three, and four-gene states of identity (except those of no pairs identical) involving any of the individuals  $A, B, C, D, E,$  and  $H$  are zero (note  $F$  and  $G$  are not included). This also means that  $F_A = F_B = F_C = F_D = F_E = F_H = 0$ . The only relations that exist are those specifically given by the pedigree.

Although the continued expansion implied above is what one would do for completeness sake, a lot of work can be avoided by doing the following. For example, a probability such as  $\gamma_{DMN}$  may be recognized as being equal to zero, because  $D$  is unrelated to the ancestors of  $M$  and  $N$  ( $E, A, B,$  and  $H$ ), which contribute genes to  $M$  and  $N$ . Therefore, since it is impossible for the three random genes, one from each of the individuals  $D, M,$  and  $N$ , to be all identical,  $\gamma_{DMN}$  may be declared to be equal to zero. Similarly, such a four-gene probability as  $\delta_{EGIL}$  (all four genes identical by descent, namely, a random gene from each of  $E, G, I,$  and  $L$ ) equals zero, because it is impossible for a random gene from  $E$  to be identical by descent to a random gene from  $G$ , having obtained its genes from only  $A$  and  $B$ . Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. I highly encourage you to follow this procedure.

Any probability which has already been worked out in my notes, you may simply substitute the numerical value and cite the particular page number, equation, and/or example number.

**Exercise 4.25.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, of which the last three generations are as follows:

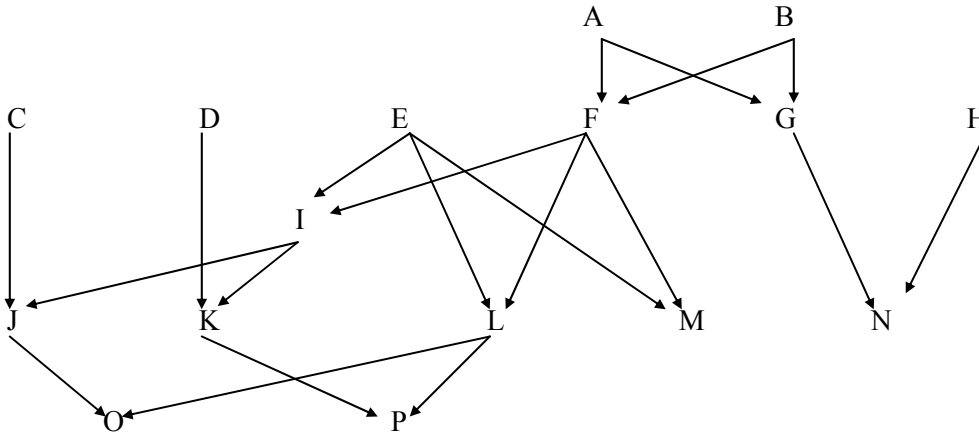


Develop a table of coefficients of coancestries between all individuals in the pedigree where the coefficients of coancestries between  $O, P, Q,$  and  $R$  are as follows (from Solution for Exercise 4.23):

	O	P	Q	R
O	9/16	7/32	15/128	15/128
P		9/16	15/128	15/128
Q			17/32	9/32
R				17/32

Express the coefficients as fractions to avoid rounding errors. What is the coefficient of inbreeding of  $U$  and  $V$ , and of the offspring between  $U$  and  $V$ ?

**Exercise 4.26.** Albert Jacquard, the Genetic Structure of Populations, p. 113, gives a complex pedigree of which the following is only a part.



a. For the generation represented by individuals  $O$  and  $P$ , calculate  $\delta_{\ddot{O}\ddot{P}}, \Delta_{\ddot{O},\ddot{P}}, \Delta_{\ddot{O}+\ddot{P}}, \gamma_{\ddot{O}P}, \gamma_{\ddot{O}\ddot{P}}, F_{\ddot{O}}, F_{\ddot{P}}$ , and  $\theta_{OP}$ , where the following probabilities for the previous generation are given.

Coefficients of coancestry ( $\theta$ ) are (see Solution for Exercise 4.23)

	J	K	L	M	N
J	1/2	1/8	1/8	1/8	1/32
K		1/2	1/8	1/8	1/32
L			1/2	1/4	1/16
M				1/2	1/16
N					1/2

The ten possible three-gene probabilities (all three genes identical) are (see Solution for Exercise 4.24)

- $\gamma_{JKL} = 1/32$                        $\gamma_{JMN} = 1/128$
- $\gamma_{JKM} = 1/32$                        $\gamma_{KLM} = 1/32$  (details not given)
- $\gamma_{JKN} = 1/128$                        $\gamma_{KLN} = 1/128$  (details not given)
- $\gamma_{JLM} = 1/32$                        $\gamma_{KMN} = 1/128$  (details not given)
- $\gamma_{JLN} = 1/128$                        $\gamma_{LMN} = 1/64$  (details not given)

The five possible four-gene probabilities (all four genes identical) are (see Solution for Exercise 4.24)

- $\delta_{JKLM} = 1/128$  (details not given)
- $\delta_{JKLN} = 1/512$  (details not given)

$$\delta_{JKMN} = 1/512$$

$$\delta_{JLMN} = 1/512$$

$$\delta_{KLMN} = 1/512 \text{ (details not given)}$$

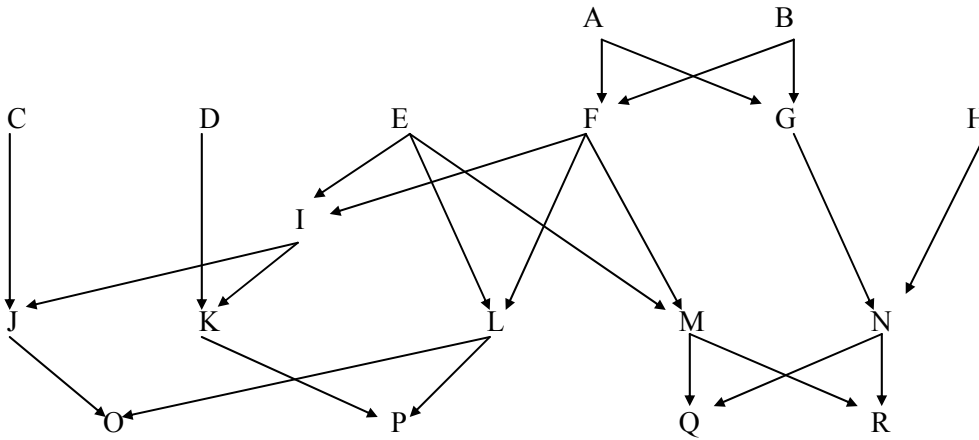
For the rules used, give its number above the term to which you apply the rule. The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Also do not forget equation (4.86) which should have been given a rule number and listed in Table 4.9.

Continue to expand backward in the pedigree only as far as necessary to establish that a given probability must be equal to zero. For example, a probability such as  $\delta_{JK\bar{L}}$  may be recognized as being equal to zero, because the two genes in  $L$  cannot be identical by descent in that  $E$  and  $F$ , the two parents of  $L$ , are unrelated. The only relations that exist are those specifically given by the pedigree. Similarly, a probability such as  $\Delta_{CD+\bar{L}}$  may be recognized as being equal to zero, because neither a random gene from  $C$  and one in  $L$  or one from  $D$  and the other one in  $L$  can be identical by descent in that  $C, D, E$ , and  $F$  are unrelated. Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. A lot of work can be avoided by adopting such a reasoning process.

Also substitute the above numerical values for  $\theta, \gamma$ , and  $\delta$  as soon as they appear in your backward expansion.

- b. Calculate the condensed coefficients of identity from the above probabilities.

**Exercise 4.27.** Albert Jacquard, *The Genetic Structure of Populations*, p. 113, gives a complex pedigree of which the following is only a part.



For the generation represented by individuals  $O, P, Q$ , and  $R$ , calculate  $\gamma_{OPQ}$ ,  $\gamma_{OPR}$ ,  $\gamma_{OQR}$ , and  $\gamma_{PQR}$  (all three genes identical by descent) and  $\delta_{OPQR}$  (all four genes identical by descent). (Note that  $\gamma_{OPQ} = \gamma_{OPR} = \gamma_{OP(MN)}$ , so you need to solve for only one.) The following probabilities for the previous generation are given.

Coefficients of coancestry ( $\theta$ ) are (see Solution for Exercise 4.23; or see Exercise 4.26)

	J	K	L	M	N
J	1/2	1/8	1/8	1/8	1/32
K		1/2	1/8	1/8	1/32
L			1/2	1/4	1/16
M				1/2	1/16
N					1/2

The ten possible three-gene probabilities (all three genes identical) are (see Solution for Exercise 4.24; or see Exercise 4.26)

$$\begin{aligned} \gamma_{JKL} &= 1/32 & \gamma_{JMN} &= 1/128 \\ \gamma_{JKM} &= 1/32 & \gamma_{KLM} &= 1/32 \text{ (details not given)} \\ \gamma_{JKN} &= 1/128 & \gamma_{KLN} &= 1/128 \text{ (details not given)} \\ \gamma_{JLM} &= 1/32 & \gamma_{KMN} &= 1/128 \text{ (details not given)} \\ \gamma_{JLN} &= 1/128 & \gamma_{LMN} &= 1/64 \text{ (details not given)} \end{aligned}$$

The five possible four-gene probabilities (all four genes identical) are (see Solution for Exercise 4.24; or see Exercise 4.26)

$$\begin{aligned} \delta_{JKLM} &= 1/128 \text{ (details not given)} \\ \delta_{JKLN} &= 1/512 \text{ (details not given)} \\ \delta_{JKMN} &= 1/512 \\ \delta_{JLMN} &= 1/512 \\ \delta_{KLMN} &= 1/512 \text{ (details not given)} \end{aligned}$$

For the rules used, give its number above the term to which you apply the rule. The rules and their numbers are listed in Table 4.9, pp. 4.104 and 4.105. Also do not forget equation (4.86) which should have been given a rule number and listed in Table 4.9.

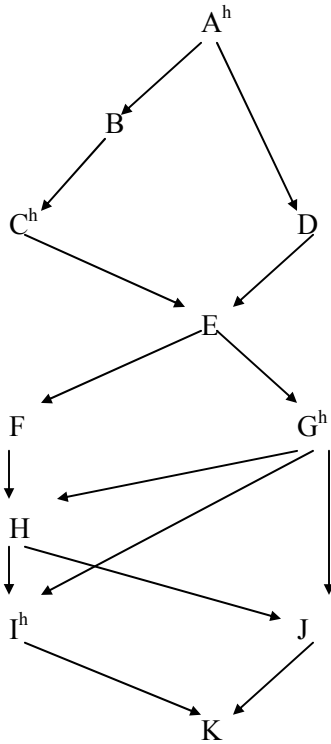
Continue to expand backward in the pedigree only as far as necessary to establish that a given probability must be equal to zero. For example, a probability such as  $\delta_{JKL}$  may be recognized as being equal to zero, because the two genes in  $L$  cannot be identical by descent in that  $E$  and  $F$ , the two parents of  $L$ , are unrelated. The only relations that exist are those specifically given by the pedigree. Similarly, a probability such as  $\Delta_{CD+L}$  may be recognized as being equal to zero, because neither a random gene from  $C$  and one in  $L$  or one from  $D$  and the other one in  $L$  can be identical by descent in that  $C, D, E$ , and  $F$  are unrelated. Indicate the setting of any such probability equal to zero by writing "P" (denoting a pedigree argument) above the term. A lot of work can be avoided by adopting such a reasoning process.

Also substitute the above numerical values for  $\theta, \gamma$  (three genes identical), and  $\delta$  (four genes identical).

**Exercise 4.28.**

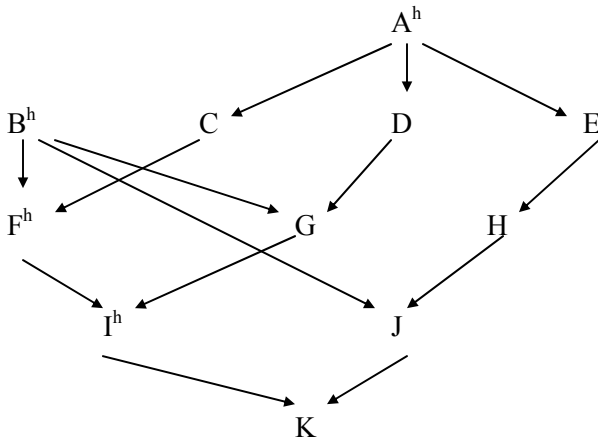
a. Define and discuss the meaning of the two-locus inbreeding coefficient. Give its probability in terms of the linkage value for self fertilization. Extend the definition to  $n$  loci. Give its probability for the case of self-fertilization (from generation 0 to 1).

b. Define and discuss the two locus, two-gamete parental descent coefficient, and the two-gamete recombinant descent coefficient. How would you extend this to  $n$  loci?

**Exercise 4.29.** Consider the following pedigree

Calculate the coefficient of inbreeding for individual  $K$  for a sex-linked locus using both of the methods presented in Sections 4.12 and 4.13. In applying the second method state what  $F_A$ , i.e.,  $F_i$  in equation (4.186), should be set equal to when the common ancestor is heterogametic as is the case in this pedigree? Give your proof or argument for what you set  $F_A$  equal to. Verify your result for the coefficient of inbreeding of  $E$ . (This is a little confusing because in reality one uses  $F_E$  in equation (4.186) and not  $F_A^h$  to obtain the coefficient of inbreeding for individual  $K$ .) (Pedigree from J.F. Crow and M. Kimura, 1970, *An Introduction to Population Genetics Theory*. Harper and Row, p. 72, has been modified.)

**Exercise 4.30.** Consider the following pedigree (After C. C. Li, 1976, First Course in Population Genetics, Fig. 18.4, p. 352.)



Calculate the coefficient of inbreeding for individual  $K$  for:

- An autosomal locus using both the tabular coancestry method (Section 4.4) and the chain of coancestry method (Section 4.5).
- A sex-linked locus using both the tabular coancestry method (Section 4.12) and the chain of coancestry method (Section 4.13).
- Compare the inbreeding values of  $K$  obtained in (a) and (b). Which one is larger? Why? Is this always true for any arbitrary pedigree?