Agronomy 615
Statistical Genetics
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SOLUTIONS TO EXERCISES FOR CHAPTER 13

Exercise 13.1.

Values for certain additional effects need to be arbitrarily assigned beyond those given in Exercise 13.1, so that the general restrictions used in Example 13.1, pp. 13.4 to 13.7, are satisfied. I assign the following: $\alpha_{B_1} = 7, \alpha_{B_2} = 3,$ and $\alpha_{B_3} = -10$, so that their sum equals zero. Concerning dominance effects, I am presently a little uncertain about how they are measured, but from my comment on p. 13.2 for the δ effect I presume that they are measured from the mean of their two corresponding homozygotes. (A rigorous discussion of multiple alleles seems to be lacking in the literature.) I assign the following dominance values: $\delta_{B_1B_2} = 1, \delta_{B_1B_3} = 2,$ and $\delta_{B_2B_3} = 3$. Finally, for the remaining additive-by-additive effects, I assign $\alpha\alpha_{A_1B_3} = 3$ and $\alpha\alpha_{A_2B_3} = -3$, so that the sum of all additive-by-additive effects equals zero. We list all effects here for convenience.

$$\mu' = 100 \qquad \alpha_{B_1} = 7 \qquad (\alpha\alpha)_{A_1B_1} = 4$$
$$\alpha_{A_1} = 9 \qquad \alpha_{B_2} = 3 \qquad (\alpha\alpha)_{A_1B_2} = 1$$
$$\alpha_{A_2} = -9 \qquad \alpha_{B_3} = -10 \qquad (\alpha\alpha)_{A_1B_3} = 3$$
$$\delta_{A_1A_2} = 7 \qquad \delta_{B_1B_2} = 1 \qquad (\alpha\alpha)_{A_2B_1} = -3$$
$$\delta_{B_1B_3} = 2 \qquad (\alpha\alpha)_{A_2B_2} = -2$$
$$\delta_{B_2B_3} = 3 \qquad (\alpha\alpha)_{A_2B_3} = -3$$

Values of double homozygotes are

$$G_{1111} = G(A_1A_1B_1B_1) = \mu' + 2\alpha_{A_1} + 2\alpha_{B_1} + 4(\alpha\alpha)_{A_1B_1}$$
$$= 100 + 2(9) + 2(7) + 4(4) = 148$$
$$G_{1122} = 100 + 2(9) + 2(3) + 4(1) = 128$$
$$G_{1133} = 100 + 2(9) + 2(-10) + 4(3) = 110$$
$$G_{2211} = 100 + 2(-9) + 2(7) + 4(-3) = 84$$
$$G_{2222} = 100 + 2(-9) + 2(3) + 4(-2) = 80$$
$$G_{2233} = 100 + 2(-9) + 2(-10) + 4(-3) = 50$$

Values of single heterozygotes are

$$G_{1112} = \mu' + 2\alpha_{A_1} + \alpha_{B_1} + \alpha_{B_2} + \delta_{B_1B_2} + 2(\alpha\alpha)_{A_1B_1} + 2(\alpha\alpha)_{A_1B_2}$$
$$= 100 + 2(9) + 7 + 3 + 1 + 2(4) + 2(1) = 139$$
$$G_{1113} = 100 + 2(9) + 7 + (-10) + 2 + 2(4) + 2(3) = 131$$
$$G_{1123} = 100 + 2(9) + 3 + (-10) + 3 + 2(1) + 2(3) = 122$$
$$G_{2212} = 100 + 2(-9) + 7 + 3 + 1 + 2(-3) + 2(-2) = 83$$
$$G_{2213} = 100 + 2(-9) + 7 + (-10) + 2 + 2(-3) + 2(-3) = 69$$
$$G_{2223} = 100 + 2(-9) + 3 + (-10) + 3 + 2(-2) + 2(-3) = 68$$
$$G_{1211} = 100 + 9 + (-9) + 7 + 2(7) + 2(4) + 2(-3) = 123$$
$$G_{1222} = 100 + 9 + (-9) + 7 + 2(3) + 2(1) + 2(-2) = 111$$
$$G_{1233} = 100 + 9 + (-9) + 7 + 2(-10) + 2(3) + 2(-3) = 87$$

Values of double heterozygotes are

$$G_{1212} = \mu' + \alpha_{A_1} + \alpha_{A_2} + \delta_{A_1A_2} + \alpha_{B_1} + \alpha_{B_2} + \delta_{B_1B_2} + (\alpha\alpha)_{A_1B_1} + (\alpha\alpha)_{A_1B_2} + (\alpha\alpha)_{A_2B_1} + (\alpha\alpha)_{A_2B_2}$$
$$= 100 + 9 + (-9) + 7 + 7 + 3 + 1 + 4 + 1 + (-3) + (-2) = 118$$
$$G_{1213} = 100 + 9 + (-9) + 7 + 7 + (-10) + 2 + 4 + 3 + (-3) + (-3) = 107$$
$$G_{1223} = 100 + 9 + (-9) + 7 + 3 + (-10) + 3 + 1 + 3 + (-2) + (-3) = 102$$

Exercise 13.2.

Since the exercise is a student proposed question, no solution is required here.