PROBLEM: Evolution of interactions – Adaptive topographies in family structured populations. Reference: Michod and Abugov Science: 210: 667-669.

genotype	f_i	df_i/dp	W_i
AA	p^2	2p	$\underset{i}{S} w_{1i}p(i/1)$
AB	2p(1-p)	2-4p	$\underset{i}{S} w_{2i}p(i/2)$
BB	$(1-p)^2$	-2+2p	$\underset{i}{S} w_{3i}p(i/3)$

Assume the additive model of interactions in which $w_{ij} = 1 + c_i + b_j$.

Also assume that selection is weak so that the adult frequencies can be approximated by the Hardy-Weinberg frequencies.

1) Calculate P(i/1) i=1,2,3 yourself. Also convince yourself that the other conditional probablilities are $P(1/2)=\frac{1}{4}p(1+p)$, $P(2/2)=\frac{1}{2}(-p^2+p+1)$, $P(3/2)=\frac{1}{4}(p-1)(p-2)$, $P(1/3)=\frac{1}{4}p^2$, $P(2/3)=p(1-\frac{1}{2}p)$, $P(3/3)=\frac{1}{4}(p-2)^2$.

2) Prove that the fitness function, F(p), equals

$$F(p) = p^2(c_1+\tfrac{1}{2}b_1-2c_2-b_2+c_3+\tfrac{1}{2}b_3) + p(2c_2+b_2-2c_3-b_3) + C$$

where C is a constant of integration.

3) Define $e_i=c_i+\frac{1}{2}b_i$ as the <u>inclusive</u> <u>fitness</u> <u>effect</u> of the i^{th} genotype when interacting with outbred sibs. Show that if you take $C=e_3$

$$F(p) = \underset{i}{S} f_i e_i \quad \text{(= mean inclusive fitness effect)}$$

4) Let us now consider altruism in which $e_1= c+\frac{1}{2}b$, $e_2=h(c+\frac{1}{2}b)$ and $e_3=0$.

The parameter h is the probability that a heterozygote performs and altruistic act. Give the conditions on c and b such that dF(p)/dp is always positive.

answer: $c/b < \frac{1}{2}$ (This is known as Hamilton's Rule for outbred sibs)