EEB 600A, Problem Set One

Due Tuesday, 4 Feb 2003

1 : Suppose you are following two species (A and B) in the fossil record, and observe that 20 percent of all your samples contain both species, while species A is found in 35 percent of the samples and species B is found in 50 percent of the samples.

(a) What is the conditional probability of finding species A when species B is present?
(b) What is the conditional probability of finding species B when species A is present?

2 : Assume that substitutions follow a Poisson process (i.e., the number follows a Poisson distribution), and that there is site-to-site variation in the sequence of interest, with fast sites having a rate of 5×10^{-7}/yr, immediately sites having a rate of 10^{-8}/yr, and slow sites having a rate of 10^{-9}/yr. The frequency of fast, medium, and slow sites are 0.05, 0.65, 0.30. If we pick a random site, what is the probability that it has not mutated after 10^7 years? *Hint:* Condition over all the possible sites.

3 : Consider the same parameters as in Problem 3, and suppose we pick a random site showing no mutation. What is the probability that this is a fast site? A medium site? A slow sight? (Hint: Bayes theorem might be of use.)

4 : Suppose the mutation rate is 10^{-6} per generation. How many generations do we have to wait to have a 50 percent chance that (at least) one mutation has occured? How many generations for a 90 percent chance?

5 : As a comparison of the binomal and Poisson distributions, suppose the mutation rate is 10^{-3} per site per year and suppose we look at 500 sites.

(a) Using the binomal, what is the probability of two or fewer mutations?
(b) What is this same probability using the Possion?

6 : Consider the following discrete random variable X, which takes on three values:

$$\Pr(X) = \begin{cases} -1 & \text{with probability } 0.1 \\ 0 & \text{with probability } 0.3 \\ 1 & \text{with probability } 0.6 \end{cases}$$

Compute the following

(a) $E[\,X\,]$
(b) $E[\,X^2\,]$
(c) $\sigma^2(X)$
(d) the Skew of X, $E[\,(X-\mu_x)^3\,]$
(e) the fourth moment of X, $E[\,(X-\mu_x)^4\,]$
(f) the kurotsis of X
(g) $E[\exp(X)\,]$