EEB 596z, Problem Set 5

Due 28 March 2002

1 : The **Weibul distribution** (after the Swedish physicist Waloddi Weibul, who proposed this distribution in 1939 for the breaking strenght of materials), has density function

$$p(x) = \lambda\, x^{\lambda-1} \exp(-x^{\lambda}) \quad \text{for} \quad x, \lambda > 0$$

[As an aside, note that the Weibul arises by assuming $y = x^{\lambda}$ follows an exponential distribution, $p(y) = \theta \exp(-\theta y)$].

Suppose we sample n values $(x_1, \cdots, x_n)$ independently from a Weibul with parameter λ.

(a) What is the resulting likelihood function $\ell(\lambda \,|\, x_1, \cdots, x_n)$, for λ?

(b) What is the resulting log-likelihood function?

(c) What is the score function for λ?

(d) What is the information (i.e., the derivate of the score function, which is the second derivative of the log-likelihood function)

(e) Suppose we observe 5 x values: 12, 15.5, 17.5, 21.3, and 13.5. Plot the log-likelihood as a function of λ (you can use Excel, or whatever program you wish).

(f) What is the MLE in this case? (You can use the previous graph)

(g) Using your results from parts (d) and (f), what is the variance of your MLE estimator?

(h) What is the probability value for the likelihood-ratio test for this data that $\lambda = 1$? For $\lambda = 4$?