EEB 596z, Problem Set Three

Due Thursday 29 Jan 2002

1 : Consider a one-way ANOVA design with 5 factors and 10 replicates per factor. Suppose that factor variance σ_t^2 is ten percent of the total variance σ_T^2 (i.e., $\sigma_t^2/\sigma_T^2 = 0.10$).

(a) Given that the total variance equals the treatment plus error variance ($\sigma_T^2 = \sigma_t^2 + \sigma_e^2$), what is σ_t^2/σ_e^2?

Note that $\sigma_T^2 = \sigma_t^2 + \sigma_e^2$ implies $\sigma_T^2/\sigma_t^2 = 1 + \sigma_e^2/\sigma_t^2$, or that $\sigma_e^2/\sigma_t^2 = \sigma_T^2/\sigma_t^2 - 1$, and hence

$$\sigma_t^2/\sigma_e^2 = \frac{1}{\sigma_T^2/\sigma_t^2 - 1} = \frac{1}{1/0.1 - 1} = \frac{1}{9} = 0.1111$$

(b) What is the 95% critical value for the F-test?

The 95% critical value f satisfies $\Pr(F_{N-1,N(n-1)} \leq f) = 0.95$. Here $N = 5, n = 10$. Using R, we find that

```
> qf(0.95, 5-1,5*(10-1))
[1] 2.578739
```

(c) What is the power of this design (assuming a test of $\alpha = 0.5$) for a fixed-effects ANOVA?

From Equation A5.23 in the notes, the power is given by

$$\Pr(F_{N-1,N(n-1),\lambda} \geq 2.578739)$$

where the noncentrality parameter $\lambda = n(N-1)\sigma_t^2/\sigma_e^2 = 10 * 4 * (1/9)$. In R, we can compute this by using

```
> 1-pf(2.578739,5-1,5*(10-1),10*4/9)
[1] 0.3207070
```

Note that we used `1-pf`, as `pf` returns the probability of being $\leq x$, while we wish the probability of being $\geq x$. Note we could also have used

```
> 1-pf(qf(0.95, 5-1,5*(10-1)),5-1,5*(10-1),10*4/9)
```

directly having R recompute the critical value.

(d) What is the power of this design under a random-effects ANOVA?

Recalling Equation A5.30b in the notes, the power is given by

$$\Pr\left[F_{N-1,N(n-1)} > \frac{F_{N-1,N(n-1),[1-\alpha]}}{1 + n\left(\sigma_t^2/\sigma_e^2\right)}\right] = \Pr\left[F_{N-1,N(n-1)} > \frac{2.578739}{1 + n\left(\sigma_t^2/\sigma_e^2\right)}\right]$$

In R,

```
> 1- pf(2.578739/(1+10/9),5-1,5*(10-1))
[1] 0.3150735
```

Giving a power close too, but slight less than, a fixed-effects design.

(e) Given these sample sizes, what is the smallest value of σ_t^2/σ_e^2 that gives a (fixed-effects) 95% test a power of 0.90? (You will need to do this, and some the remaining problems, by trial and error.)

We try various values of σ_t^2/σ_e^2:

```
> 1-pf(2.578739,5-1,5*(10-1),10*4*(1/2)) # trying 1/2
[1] 0.943019
1-pf(2.578739,5-1,5*(10-1),10*4*(0.4)) # trying 0.4
```

```
[1] 0.8768408
1-pf(2.578739,5-1,5*(10-1),10*4*(0.43)) # trying 0.43
[1] 0.9015374
```

Hence $\sigma_t^2/\sigma_e^2 = 0.43$, or (rearraning results from (a)),

$$\sigma_t^2/\sigma_T^2 = \frac{1}{1+1/(\sigma_t^2/\sigma_e^2)} = \frac{1}{1+1/0.43} = 0.30$$

Hence, this design has the power to detect an effect accounting for 30% or more of the total variation.

Note that we could also solve this more quickly by using the graphics commands in R, plotting power as a function of varying values of σ_t^2/σ_e^2. In particular to example power for $0.1 \leq \sigma_t^2/\sigma_e^2 \leq 0.9$, the R code is

```
> curve(1-pf(2.578739,5-1,5*(10-1),10*4*x), 0.1,0.9)
```

which returns a nice graph. We can focus in on finer regions of the cruve by restircting the interval to smaller regions, e.g.,

```
> curve(1-pf(2.578739,5-1,5*(10-1),10*4*x), 0.4,0.5)
```

(f) Given these sample sizes, what is the smallest value of σ_t^2/σ_e^2 that gives a random-effects 95% test a power of 0.90?

Here, we need to solve for σ_t^2/σ_e^2 in

$$\Pr\left[F_{N-1,N(n-1)} > \frac{2.578739}{1+n\,(\sigma_t^2/\sigma_e^2)}\right] = 0.9$$

Using R [`qf(0.1,5-1,5*(10-1))`], we find that $\Pr\left[F_{N-1,N(n-1)} > 0.26323\right] = 0.9$, hence we solve for

$$\frac{2.578739}{1+10\,(\sigma_t^2/\sigma_e^2)} = 0.26323$$

giving = $\sigma_t^2/\sigma_e^2 = 0.8796524$, so that σ_t^2 must account for at least 46.8% of the total variance (using the expression in part (f) to covert σ_t^2/σ_e^2 into σ_t^2/σ_T^2.)

(g) How many replicates per factor are needed to give the fixed-effects ANOVA a power of 90% under a test of significant with $\alpha = 0.05$?

Once again, we can use trail and error, varying n to solve

$$\Pr(F_{5-1,5(n-1),n(5-1)/9} \geq F_{5-1,5(n-1),[0.95]}) = 0.9$$

in R, we first try $n = 30$,

```
> n <- 30; 1-pf(qf(0.95, 5-1,5*(n-1)),5-1,5*(n-1),n*4/9)
[1] 0.8339192
n <- 35; 1-pf(qf(0.95, 5-1,5*(n-1)),5-1,5*(n-1),n*4/9)
[1] 0.8940105
n <- 36; 1-pf(qf(0.95, 5-1,5*(n-1)),5-1,5*(n-1),n*4/9)
[1] 0.9034603
```

$n = 36$ it is.

(h) How many replicates per factor are needed to give the random-effects ANOVA a power of 90% under a test of significant with $\alpha = 0.05$? (Again, need to use trail and error)

Here we need to find n such that

$$\Pr\left[F_{5-1,5(n-1)} > \frac{F_{5-1,5(n-1),[0.95]}}{1+n/9}\right] = 0.9$$

in R, we first try $n = 80$,

```
n <- 80; 1-pf(qf(0.95, 5-1,5*(n-1))/(1+n/9),5-1,5*(n-1))
[1] 0.914335
> n <- 73; 1-pf(qf(0.95, 5-1,5*(n-1))/(1+n/9),5-1,5*(n-1))
[1] 0.9015613
> n <- 72; 1-pf(qf(0.95, 5-1,5*(n-1))/(1+n/9),5-1,5*(n-1))
[1] 0.8995078
```

giving $n = 73$.