Power Calculations in R

The notes supplement both the "Introduction to R" notes and the notes of Computing power. To remind the reader how to compute p and critical values for the distributions useful in power calculations (normal, t, χ^2, F), here are a number of examples. The reader should also work through these examples in **R**.

The **>** indicates the **R** prompt, [**1**] the output from **R**

Normals

• Compute the 99% **quantile** (the value correspond to a the 99% probability) for a unit normal.

```
> qnorm(0.99)
[1] 2.326348
```

Hence, $\Pr(U \leq 2.326348) = 0.99$

We used the notation $z_{(\alpha)}$ in the power notes (eq., Equations A5.1a-c) as the solution to $\Pr(U \leq z_{(\alpha)}) = \alpha$ where U is the unit normal. This is just the α quantile. For example, to compute $z_{(0.01)}$,

```
> qnorm(0.01)
[1] -2.326348
```

Hence, $\Pr(U \leq -2.326348) = 0.01$

• Compute the probability that a unit normal is less than or equal to -0.35.

```
> pnorm(-0.35)
[1] 0.3631693
```

Hence, $\Pr(U \leq -0.35) = 0.3631693$

Note that **q** indicates computing a quantile (given a probability, **R** returns a number), while **p** computes a probability (given a value, it returns the probability of being less that or equal to that value)

Student's t

• Compute the 95% quantile for a t distribution with 19 degrees of freedom.

```
> qt(0.95,19)
[1] 1.729133
```

Hence, $\Pr(t_{19} \leq 1.729133) = 0.95$

• Compute the probability that a t random variable with 6 degrees of freedom is greater than 1.1

```
> 1-pt(1.1,6)
[1] 0.1567481
```

Hence, $\Pr(t_6 > 1.1) = 1 - \Pr(t_6 \leq 1.1) = 0.1567481$

χ^2 distribution

• Compute the 90% quantile for a (central) χ^2 distribution for 15 degrees of freedom

```
> qchisq(0.9,15)
[1] 22.30713
```

Hence, $\Pr(\chi^2_{15} \leq 22.30713) = 0.9$

• Compute probability that a (central) χ^2 distribution with 13 degrees of freedom is less than or equal to 21.

```
> pchisq(21,13)
[1] 0.9270714
```

Hence, $\Pr(\chi^2_{13} \leq 21) = 0.9270714$

• Compute probability that a χ^2 distribution with 13 degrees of freedom and noncentrality parameter 5.4 is less than or equal to 21.

```
> pchisq(21,13,5.4)
[1] 0.6837649
```

Hence, $\Pr(\chi^2_{13,5.4} \leq 21) = 0.6837649$

• Compute the 50% quantile for a χ^2 distribution with 7 degrees of freedom and noncentrality parameter 3.

```
> qchisq(0.5,7,3)
[1] 9.180148
```

Hence, $\Pr(\chi^2_{7,3} \leq 9.180148) = 0.5$

F distribution

• Compute the value C for an F distribution with 3 and 16 degrees of freedom such that $\Pr(F_{3,16} > C) = 0.05$ (This value is used in Example 3 in the power notes)

```
> qf(1-0.05,3,16)
[1] 3.238872
```

Hence, $\Pr(F_{3,16} > 3.238872) = 1 - \Pr(F_{3,16} \geq 3.238872) = 1 - 0.95 = 0.05$

• Compute the probability that a noncentral F with 3 and 16 degrees of freedom and noncentality parameter 5 exceeds 3.24 (from Example 3)

```
> 1-pf(3.24,3,16,5)
[1] 0.3533815
```

Hence, $\Pr(F_{3,16,5} > 3.24) = 1 - \Pr(F_{3,16,5} \geq 3.24) = 0.3533815$

Plotting Power Curves

Using some of the graphics commands introduced in the Introduction to **R** Notes, we can compute various power curves. Be sure and do the graphics examples from the Introduction Notes first. Some other useful coding in **R** are as follows:

• **sqrt(x)** – returns $\sqrt{x}$

• Defining functions. Suppose we wish to define $g(x, y, z) = log(x + y)/z$. In **R** we do this by using the syntax

```
> g <- function(x,y,z) log(x+y)/z
```

For example, consider Equation A5.2b in the power notes, the α-level critical value when the mean is μ_0, the variance is $\sigma_0^2 = f_0^2/n$, $T_c(\alpha) = \mu_0 + \sigma_0 z_{(1-\alpha)}$. In **R** we can write this function as

```
Talpha <- function(mu0,f0,n,alpha)* mu0
+ sqrt(f0*f0/n)*qnorm(1-alpha)
```

To compute the critical value for $\alpha = 0.01$, $\mu_0 = 12$, $f_0 = 3$, and $n = 25$,

```
> Talpha(12,3,25,0.01)
[1] 12.80587
```

Likewise, if the true mean is μ_1 and the true variance is $\sigma_1^2 = f_1^2/n$, the power for an α-level test is given from Equation A5.3, $\Pr(U > [T_c(\alpha) - \mu)1]/\sigma_1)$, and we can use the previous function write this in **R** as the function

```
    Palpha <- function(mu0,mu1,f0,f1,n,alpha)
1-pnorm( (Talpha(mu0,f0,n,alpha)-mu1)/sqrt(f1*f12/n))
```

We can this use to plot various power functions. For example, how does power change as we change the true mean? Suppose $\mu_0 = 5, f_0 = 3, f_1 = 2, n = 50$ and $\alpha = 0.1$. To plot the power for μ_1 ranging from 0 to 15, in **R** use the above function and type

```
> curve(Palpha(5,x,3,2,50,0.1), 0,15)
```

Note that the syntax for the curve function is to use x as the variable to change, so that **curve(f(x),a,b)** plots how f(x) changes from x=a to x=b.

To see how power changes as we vary n, assume that $\mu_1 = 6$, and vary n from 20 to 300

```
> curve(Palpha(5,6,3,2,x,0.1), 20,300)
```

Iterative Power Calculations

As a final example of computing power, consider Example 3 in the Power notes, on computing the power for an ANOVA. In this example there are $N = 4$ treatments that account for 1/3 of all variance. As shown in the example, with n replicates per treatment, this implies the noncentrality parameter of the F distribution is $= (N - 1)n(1/3) = n$. We increase power by increasing the number of replicates per treatment n, but this also requires us to recompute the critical value (which also changes with n). As we increase n, the F degrees of freedom become 3 and $4(n - 1)$. The 95% critical value for any n can be computed in **R** by defining the function

```
Fcrit <- function (n) qf(0.95,3,4*(n-1))
```

For example,

```
> Fcrit(5)
[1] 3.238872      (the value in Example for n=5)
> Fcrit(15)
[1] 2.769431      (the value in Example for n=15)
```

To compute the power for various values of n, use the above function **`Fcrit`** and define the function **`Fpower`** (or whatever you wish to call it) by

```
> Fpower <- function(n) 1-pf(Fcrit(n),3,4*(n-1),n)
> Fpower(5)
[1] 0.3535594        (the power for n=5)
> Fpower(15)
[1] 0.8957212        (the power for n =15)
> Fpower(16)
[1] 0.9167217        (the power for n=16)
```