Power Calculations in R

The notes supplement both the “Introduction to R” notes and the notes of Computing power. To remind the reader how to compute \( p \) and critical values for the distributions useful in power calculations (normal, \( t \), \( \chi^2 \), \( F \)), here are a number of examples. The reader should also work through these examples in R.

The > indicates the R prompt. [1] the output from R

Normals

• Compute the 99% quantile (the value correspond to a the 99% probability) for a unit normal.

> qnorm(0.99)
[1] 2.326348
Hence, \( \Pr(U \leq 2.326348) = 0.99\)

We used the notation \( z(\alpha) \) in the power notes (eq., Equations A5.1a-c) as the solution to \( \Pr(U \leq z(\alpha)) = \alpha \) where \( U \) is the unit normal. This is just the \( \alpha \) quantile. For example, to compute \( z(0.01)\),

> qnorm(0.01)
[1] -2.326348
Hence, \( \Pr(U \leq -2.326348) = 0.01\)

• Compute the probability that a unit normal is less than or equal to -0.35.

> pnorm(-0.35)
[1] 0.3631693
Hence, \( \Pr(U \leq -0.35) = 0.3631693\)

Note that q indicates computing a quantile (given a probability, R returns a number), while p computes a probability (given a value, it returns the probability of being less that or equal to that value)

Student’s t

• Compute the 95% quantile for a \( t \) distribution with 19 degrees of freedom.

> qt(0.95, 19)
[1] 1.729133
Hence, \( \Pr(t_{19} \leq 1.729133) = 0.95\)

• Compute the probability that a \( t \) random variable with 6 degrees of freedom is greater than 1.1

> 1-pf(1.1, 6)
[1] 0.1567481
Hence, \( \Pr(t_6 > 1.1) = 1 - \Pr(t_6 \leq 1.1) = 0.1567481\)
\( \chi^2 \) distribution

- Compute the 90\% quantile for a (central) \( \chi^2 \) distribution for 15 degrees of freedom
  
  \[
  > \text{qchisq}(0.9, 15) \\
  \text{[1]} \ 22.30713 \\
  \text{Hence, } \Pr(\chi^2_{15} \leq 22.30713) = 0.9
  \]

- Compute probability that a (central) \( \chi^2 \) distribution with 13 degrees of freedom is less than or equal to 21.
  
  \[
  > \text{pchisq}(21, 13) \\
  \text{[1]} \ 0.9270714 \\
  \text{Hence, } \Pr(\chi^2_{13} \leq 21) = 0.9270714
  \]

- Compute probability that a \( \chi^2 \) distribution with 13 degrees of freedom and noncentrality parameter 5.4 is less than or equal to 21.
  
  \[
  > \text{pchisq}(21, 13, 5.4) \\
  \text{[1]} \ 0.6837649 \\
  \text{Hence, } \Pr(\chi^2_{13, 5.4} \leq 21) = 0.6837649
  \]

- Compute the 50\% quantile for a \( \chi^2 \) distribution with 7 degrees of freedom and noncentrality parameter 3.
  
  \[
  > \text{qchisq}(0.5, 7, 3) \\
  \text{[1]} \ 9.180148 \\
  \text{Hence, } \Pr(\chi^2_{7, 3} \leq 9.180148) = 0.5
  \]

\( F \) distribution

- Compute the value \( C \) for an \( F \) distribution with 3 and 16 degrees of freedom such that \( \Pr(F_{3,16} > C) = 0.05 \) (This value is used in Example 3 in the power notes)
  
  \[
  > \text{qf}(1-0.05, 3, 16) \\
  \text{[1]} \ 3.238872 \\
  \text{Hence, } \Pr(F_{3,16} > 3.238872) = 1 - \Pr(F_{3,16} \geq 3.238872) = 1 - 0.95 = 0.05
  \]

- Compute the probability that a noncentral \( F \) with 3 and 16 degrees of freedom and noncentrality parameter 5 exceeds 3.24 (from Example 3)
  
  \[
  > 1-\text{pf}(3.24, 3, 16, 5) \\
  \text{[1]} \ 0.3533815 \\
  \text{Hence, } \Pr(F_{3,16, 5} > 3.24) = 1 - \Pr(F_{3,16, 5} \geq 3.24) = 0.3533815
  \]

Plotting Power Curves

Using some of the graphics commands introduced in the Introduction to \texttt{R} Notes, we can compute various power curves. Be sure and do the graphics examples from the Introduction Notes first. Some other useful coding in \texttt{R} are as follows:
• \texttt{sqrt(x)} – returns $\sqrt{x}$

Defining functions. Suppose we wish to define $g(x, y, z) = \log(x + y)/z$. In R we do this by using the syntax

\begin{verbatim}
> g <- function(x,y,z) log(x+y)/z
\end{verbatim}

For example, consider Equation A5.2b in the power notes, the \( \alpha \)-level critical value when the mean is \( \mu_0 \), the variance is \( \sigma^2_0 = f_0^2/n \), \( T_c(\alpha) = \mu_0 + \sigma_0 z_{1-\alpha} \). In R we can write this function as

\begin{verbatim}
Talpha <- function(mu0,f0,n,alpha)* mu0 + sqrt(f0*f0/n)*qnorm(1-alpha)
\end{verbatim}

To compute the critical value for \( \alpha = 0.01 \), \( \mu_0 = 12 \), \( f_0 = 3 \), and \( n = 25 \),

\begin{verbatim}
> Talpha(12,3,25,0.01)
[1] 12.80587
\end{verbatim}

Likewise, if the true mean is \( \mu_1 \) and the true variance is \( \sigma^2_1 = f_1^2/n \), the power for an \( \alpha \)-level test is given from Equation A5.3, \( \Pr(U > [T_c(\alpha) - \mu]/\sigma_1) \), and we can use the previous function write this in R as the function

\begin{verbatim}
Palpha <- function(mu0,mu1,f0,f1,n,alpha) 1-pnorm( (Talpha(mu0,f0,n,alpha)-mu1)/sqrt(f1*f12/n))
\end{verbatim}

We can this use to plot various power functions. For example, how does power change as we change the true mean? Suppose \( \mu_0 = 5 \), \( f_0 = 3 \), \( f_1 = 2 \), \( n = 50 \) and \( \alpha = 0.1 \). To plot the power for \( \mu_1 \) ranging from 0 to 15, in R use the above function and type

\begin{verbatim}
> curve(Palpha(5,x,3,2,50,0.1), 0,15)
\end{verbatim}

Iterative Power Calculations

As a final example of computing power, consider Example 3 in the Power notes, on computing the power for an ANOVA. In this example there are \( N = 4 \) treatments that account for \( 1/3 \) of all variance. As shown in the example, with \( n \) replicates per treatment, this implies the noncentrality parameter of the \( F \) distribution is \( = (N - 1)n(1/3) = n \). We increase power by increasing the number of replicates per treatment \( n \), but this also requires us to recompute the critical value (which also changes with \( n \)). As we increase \( n \), the \( F \) degrees of freedom become \( 3 \) and \( 4(n - 1) \). The 95\% critical value for any \( n \) can be computed in R by defining the function

\begin{verbatim}
Fcrit <- function (n) qf(0.95,3,4*(n-1))
\end{verbatim}
For example,

```r
> Fcrit(5)
[1] 3.238872  (the value in Example for n=5)
> Fcrit(15)
[1] 2.769431  (the value in Example for n=15)
```

To compute the power for various values of \( n \), use the above function `Fcrit` and define the function `Fpower` (or whatever you wish to call it) by

```r
> Fpower <- function(n) 1 - pf(Fcrit(n), 3, 4*(n-1), n)
> Fpower(5)
[1] 0.3535594  (the power for n=5)
> Fpower(15)
[1] 0.8957212  (the power for n =15)
> Fpower(16)
[1] 0.9167217  (the power for n=16)
```