EEB 581, Problem Set Nine , Due 4 April 2006

Some helpful **R** commands before starting!

• `t(A)` computes the transpose of A.

• `det(A)` computes the determinant of A.

• `diag(c)` takes an n-dimensional column vector c creates a diagonal matrix whose ith diagonal element is the ith element of c.

•`eigen(A)` computes the eigenvalues (as a row vector, with the ith element the ith eigenvalue) and eigenvectors (as a matrix, whose i-th colunm is the i-th eigenvector) of A.

•`eigen(A)$values` directly returns the row vector of eigenvalues, so that `eval<- eigen(A)$values` stores this vector for future use.

• Note that if b is a vector, then the **R** command `b∧5` returns a vector where every element has been raised to the 5-th power.

•`eigen(A)$vectors` directly returns the matrix of eigenvectors.

1 : Consider the following covariance matrix,

$$A = \begin{pmatrix} 40 & 30 & -30 \\ 30 & 40 & -20 \\ -30 & -20 & 40 \end{pmatrix}$$

(a) What is the trace of $\mathbf{A}$?.

(b) Use **R** to compute the determinant of $\mathbf{A}$.

(c) Use **R** to solve for eigenvalues and the corresponding eigenvectors.

(d) Use **R** to show that the eigenvectors and their corresponding eigenvalues obtained in (c) satisfy the eigenvalue-eigenvector equation (i.e., that $\mathbf{A}\mathbf{e_i} = \lambda_i \mathbf{e_i}$).

(e) Using your results from (a) and (c), is trace ($\mathbf{A}$) = sum of the eigenvalues?

(f) Using your results from (a) and (c), is det($\mathbf{A}$) = product of the eigenvalues?

(g) Diagonalize $\mathbf{A}$, i.e., write it as the matrix product $\mathbf{U}\,\Lambda\mathbf{U}^T$.

(h) Use matrix multiplication in **R** to show indeed that $\mathbf{A} = \mathbf{U}\,\Lambda\mathbf{U}^T$.

(i) Use matrix multiplication in **R** to show that $\mathbf{U}^T\mathbf{U} = \mathbf{U}\mathbf{U}^T = \mathbf{I}$.

(j) What are the eigenvalues for $\mathbf{A}^3$? $\mathbf{A}^{-1}$? $\mathbf{A}^{1/2}$? $\mathbf{A}^{-1/2}$?

(k) Use diagonalization to compute these four matrices.

(l) Use **R** to show that $\mathbf{A}^{1/2}\mathbf{A}^{1/2}$=$\mathbf{A}$, with $\mathbf{A}$ obtained as in (j).

(m) For this covariance matrix (which is the vector $\mathbf{x}^T = (x_1, x_2, x_3)$, which linear combination of the x_i corresponds to PC 1? How much of the total variation does PC 1 explain?

(n) What linear combination gives PC 2? How much of the total variation does PC 2 explain?