EEB 581, Problem Set Seven

Solutions

1 : Consider following system of equations:

$$4x_1 + 3x_2 + 6x_3 = 6$$
$$2x_1 + 6x_2 + 2x_3 = 4$$

(a) Write this in matrix form, $\mathbf{Ax} = \mathbf{y}$.

$$\mathbf{A} = \begin{pmatrix} 4 & 3 & 6 \\ 2 & 6 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 6 \\ 4 \end{pmatrix},$$

(b) Compute a generalized inverse $\mathbf{A}^-$ of $\mathbf{A}$.

In **R**

```
> library(MASS)
> A <- matrix(c(4,2,3,6,6,2),nrow=2)
> gA <- ginv(A)
```

typing **gA** returns

```
            [,1]        [,2]
[1,]  0.08064516  -0.02419355
[2,] -0.07741935   0.20322581
[3,]  0.15161290  -0.08548387
```

(c) Recalling that a g-inverse satisfies $\mathbf{AA}^-\mathbf{A}$=$\mathbf{A}$, use $\mathbf{R}$ to compute $\mathbf{AA}^-\mathbf{A}$. Does this equal $\mathbf{A}$?

```
> A%*%gA%*%A
```

R returns

```
      [,1]  [,2]  [,3]
[1,]   4     3     6
[2,]   2     6     2
```

(d) What is one solution to these equation (e.g., compute $\mathbf{x}$ = $\mathbf{A}^-\mathbf{y}$).

```
   > y <- matrix(c(6,4),nrow=2)
> gA%*%y  R returns
            [,1]
[1,]  0.3870968
[2,]  0.3483871
[3,]  0.5677419
```

(e) Recall that a consistent system of equations satisfies $\mathbf{AA}^-\mathbf{y}$ = $\mathbf{y}$. Is our system consistent?

```
   >A%*% gA%*%y
```

R returns

```
      [,1]
[1,]   6
[2,]   4
```

(f) Use **R** to compute $\mathbf{I}$ - $\mathbf{A}^{-}\mathbf{A}$

```
   > I <- matrix(c(1,0,0,0,1,0,0,0,1),nrow=3)
> I - gA% *% A
```

R returns

```
           [,1]         [,2]        [,3]
[1,]   0.725806  -0.09677419  -0.43548387
[2,]  -0.0967742  0.01290323   0.05806452
[3,]  -0.4354839  0.05806452   0.26129032
```

(g) Recall that for a consistent system, all solutions can be written as $\mathbf{x} = \mathbf{A}^{-}\mathbf{y} + (\mathbf{I} - \mathbf{A}^{-}\mathbf{A})\,\mathbf{c}$, where $\mathbf{c}$ is any vector of constants. What is the family of solutions for this equation?

$$x_1 = 0.387 + 0.726c_1 - 0.097c_2 - 0.435c_3$$
$$x_2 = 0.348 - 0.097c_1 + 0.013c_2 + 0.058c_3$$
$$x_3 = 0.568 - 0.435c_1 + 0.058c_2 + 0.261c_3$$