EEB 581, Problem Set Five

Due Tuesday, 28 Feb. 2004

1 : Consider the matrix $\mathbf{A}$ and vector $\mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} 1 & 3 \\ 3 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

By hand, compute the following (its OK to check the results in R):

(a) $\mathbf{Ab}$
(b) $\mathbf{b}^T\mathbf{A}$
(c) $\mathbf{b}^T\mathbf{b}$
(d) $\mathbf{b}\mathbf{b}^T$
(e) det($\mathbf{A}$)
(f) $\mathbf{A}^{-1}$
(g) $\mathbf{A}^T$

2 : Consider the following set of equations

$$\begin{aligned} 8x_1 + 13x_2 - 4x_3 + x_4 &= 9 \\ -4x_1 + x_2 + 5x_3 - 3x_4 &= 5 \\ 7x_1 + 9x_2 + 2x_3 + 7x_4 &= -4 \\ 3x_1 + 4x_2 + 6x_3 + 2x_4 &= 12 \end{aligned}$$

(a) Express this system of equations in matrix form
(b) Solve for the vector of the unknown x_i (ok to use **R** here!). here!).

3 : Suppose the vector x_1, x_2, x_3 has covariance matrix

$$\mathbf{V} = \begin{pmatrix} 12 & 1 & -3 \\ 1 & 10 & -2 \\ -3 & -2 & 50 \end{pmatrix}$$

Consider two new random variables,

$$y = 3x_1 + 2x_2 - 5x_3, \qquad z = x_1 - 12x_2 + 2x_3$$

(a) Compute $\sigma^2(y)$. (OK to use **R** throughout)
(b) Compute $\sigma^2(z)$.
(c) Compute the correlation between y and z — recall $\rho(y,z) = \sigma(y,z)/\sqrt{\sigma^2(y) \cdot \sigma^2(z)}$

4 : Suppose x_1, x_2, and x_3 are multivariate normally distributed with means $\mu_1 = 1$, $\mu_2 = 0$, $\mu_3 = -2$, and covariance structure

$$\sigma^2(x_1) = 3, \quad \sigma^2(x_2) = 4, \quad \sigma^2(x_3) = 6, \quad \sigma(x_1, x_2) = 1, \quad \sigma(x_1, x_3) = -1, \quad \sigma(x_2, x_3) = 2$$

Finally, define $y = x_1 - 3x_2 + 4x_3$ and $z = 3x_1 + 4x_2 - 7x_3$.

(a) Compute $\sigma^2(y)$.
(b) Compute $\sigma^2(z)$.
(c) Compute $\sigma^2(y,z)$, $\rho(y,z)$
(d) What is the distribution of x_1, x_2 given x_3?
(e) What is the regression of x_1 on x_2 and x_3?
(f) What is the conditional variance of x_1 given x_2 and x_3?