EEB 596z, Problem Set Four

Due Tuesday 16 Feb 2006

1 : Consider a one-way ANOVA design with 5 factors and 10 replicates per factor. Suppose that factor variance σ_τ^2 is ten percent of the total variance σ_T^2 (i.e., $\sigma_\tau^2/\sigma_T^2 = 0.10$).

(a) Given that the total variance equals the treatment plus error variance ($\sigma_T^2 = \sigma_\tau^2 + \sigma_e^2$), what is σ_τ^2/σ_e^2?

(b) What is the 95% critical value for the F-test?

(c) What is the power of this design (assuming a test of $\alpha = 0.05$) for a fixed-effects ANOVA?

(d) What is the power of this design under a random-effects ANOVA?

(e) Given these sample sizes, what is the smallest value of σ_τ^2/σ_e^2 that gives a (fixed-effects) 95% test a power of 0.90? (You will need to do this, and some the remaining problems, by trial and error.)

(f) Given these sample sizes, what is the smallest value of σ_τ^2/σ_e^2 that gives a random-effects 95% test a power of 0.90?

(g) How many replicates per factor are needed to give the fixed-effects ANOVA a power of 90% under a test of significant with $\alpha = 0.05$?

(h) How many replicates per factor are needed to give the random-effects ANOVA a power of 90% under a test of significant with $\alpha = 0.05$?

2 : Optimal design for a random-effects ANOVA. Suppose you have a total $T = 100$ measurements that you can make, and you have to decide how best to allocate them over N and n in a random-effects design. Should one chose more factors (increase N) at the expense of fewer replicates n per factor? Obviously, there is some intermediate trade-off between the two. Suppose that the factor variance is $\sigma_\tau^2 = 10$ and the error variance $\sigma_e^2 = 20$.

(a) Compute the power of this design for the following combinations of N and n:

50,2 33,3 25,4 20,5 10,10 5,20 4,25 3,33 2,50

Hint: It might make sense to first write an R function to do this for arbitary N, n

(b) What is the optimal design (i.e., which combination of N and n gives the largest power)?

(c) Repeat (a) and (b) assuming $\sigma_\tau^2 = 20$, $\sigma_e^2 = 10$