

EEB 581, Problem Set Two

Due 31 January 2006

Consider the following data, assuming the  $x_i$  are iid normals.

$$\bar{x} = 10, \quad \sum_{i=1}^{15} (x_i - \bar{x})^2 = 100, \quad n = 15$$

- 1: Use  $\mathbf{R}$  to find the following:
  - (i)  $X$  such that  $\Pr(U \leq X) = 0.25$
  - (ii)  $X$  such that  $\Pr(U \leq X) = 0.75$
  - (iii)  $X$  such that  $\Pr(U \leq X) = 0.005$
  - (iv)  $X$  such that  $\Pr(U \leq X) = 0.995$
  - (v) Redo i - iv replacing the Unit normal  $U$  with  $t_{14}$ , a  $t$ -distribution with 14 df.
- 2: Suppose we know that the true variance for  $x$  is  $\sigma_x^2 = 4$ . Compute 50% and 99% confidence intervals for the true mean  $\mu_x$ . (Hint: Results from Problem 1 may be useful!)
- 3: Redo Problem 2, but now assume that the variance is estimated (and hence a  $t$ -distribution is needed).
- 4: Assuming the appropriate normality assumptions, compute the 95% confidence interval for  $\sigma_x^2$ .
- 5: Under the null hypothesis that the true variance is 20, how likely are we to observe a value as small as the observed sample variance (which is  $100/14 = 7.14$ )? Hint: Express this as a  $\chi^2$  probability, as we want to find  $\Pr(\text{Var}(x) \leq 7.14)$ .