1: Data was measured on 50 individuals for arm size \((x)\) and brain size \((y)\), with the following results:

\[
\bar{x} = 10, \quad \bar{y} = 50, \quad \sum_{i=1}^{50} (x_i - \bar{x})^2 = 100, \quad \sum_{i=1}^{50} (y_i - \bar{y})^2 = 400, \quad \sum_{i=1}^{50} (x_i - \bar{x})(y_i - \bar{y}) = 175
\]

(a) Compute the variances of \(x\) and \(y\), their covariance, and their correlation.

\[
\text{Var}(x) = \frac{100}{49} = 2.04, \quad \text{Var}(y) = \frac{400}{49} = 8.16, \quad \text{Cov}(x, y) = \frac{175}{49} = 3.57
\]

\[
\text{Corr}(x, y) = \frac{3.57}{\sqrt{2.04} \cdot \sqrt{8.16}} = 0.88
\]

(b) What the best linear regression of arm size \((x)\) on brain size \((y)\)?

\[
b_{x|y} = \frac{3.57}{8.16} = 0.44, \quad a = \bar{x} - b_{x|y} \bar{y} = 10 - 0.44 \cdot 50 = -11.88
\]

Hence, the regression is (Arm size) = -11.88+ 0.44(Brain size)

(c) What the best linear regression of brain size \((y)\) on arm size \((x)\)?

\[
b_{y|x} = \frac{3.57}{2.04} = 1.75, \quad a = \bar{y} - b_{y|x} \bar{x} = 50 - 1.75 \cdot 10 = 32.58
\]

Hence, the regression is (Brain size) = 32.50 + 1.75(Arm size)

(d) What fraction of the total variance in brain size does the regression account for?

\[
\text{Fraction of the total variance explained by the regression is the squared correlation, or}
\]

\[
0.88^2 = 0.766
\]

2: What is the covariance between a particular data point \((x_i)\) and the sample mean \(\bar{x}\)?

\[
\sigma \left( \frac{1}{n} \sum_{j=1}^{n} x_j \right) = \frac{1}{n} \sum_{j=1}^{n} \sigma (x_i, x_j) = \frac{1}{n} \sum_{j=1}^{n} \sigma (x_i, x_j) = \frac{1}{n} \sum_{j \neq i}^{n} \sigma (x_i, x_j) = \frac{\sigma^2 (x_i)}{n}
\]