

EEB 581, Problem Set Seven

Due Tuesday, 30 March 2004

1 : The Weibul distribution (after the Swedish physicist Waloddi Weibul, who proposed the distribution in 1939 for the breaking strenght of materials), has density function

$$p(x) = \lambda x^{\lambda-1} \exp(-x^\lambda) \quad \text{for } x, \lambda > 0$$

[As an aside, note that the Weibul arises by assuming $y = x^\lambda$ follows an exponential distribution, $p(y) = \theta \exp(-\theta y)$].

- (a) What is the resulting likelihood function $\ell(\lambda | x_1, \dots, x_n)$, for λ ?
- (b) What is the resulting log-likelihood function?
- (c) What is the score function? (Helpful hint, $dx/d\lambda = x \ln(x)$).
- (d) What is the second derivative of the log-likelihood function?
- (e) Suppose 5 values, 0.10, 0.25, 0.5, 1, and 2 are observed. Plot the resulting log-likelihood function
- (f) What is the approximate sample variance?
- (g) What is an approximate 95% confidence interval for λ ?
- (h) What is the p value for a likelihood ratio test that $\lambda = 0.5$? $\lambda = 1$?