

EEB 581, Problem Set Six

Solutions

1 : Consider following system of equations:

$$4x_1 + 3x_2 + 6x_3 = 6$$

$$2x_1 + 6x_2 + 2x_3 = 4$$

(a) Write this in matrix form, $\mathbf{Ax} = \mathbf{y}$.

$$\mathbf{A} = \begin{pmatrix} 4 & 3 & 6 \\ 2 & 6 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} 6 \\ 4 \end{pmatrix},$$

(b) Compute a generalized inverse \mathbf{A}^- of \mathbf{A} .

In R

```
> library(MASS)
> A <- matrix(c(4,2,3,6,6,2),nrow=2)
> gA <- ginv(A)
typing gA returns
```

```
      [,1]      [,2]
[1,] 0.08064516 -0.02419355
[2,] -0.07741935  0.20322581
[3,] 0.15161290 -0.08548387
```

(c) Recalling that a g-inverse satisfies $\mathbf{AA}^- \mathbf{A} = \mathbf{A}$, use R to compute $\mathbf{AA}^- \mathbf{A}$. Does this equal \mathbf{A} ?

```
> A**gA**A
```

R returns

```
      [,1] [,2] [,3]
[1,]  4    3    6
[2,]  2    6    2
```

(d) What is one solution to these equation (e.g., compute $\mathbf{x} = \mathbf{A}^- \mathbf{y}$).

```
> y <- matrix(c(6,4),nrow=2)
```

```
> gA**y R returns
```

```
      [,1]
[1,] 0.3870968
[2,] 0.3483871
[3,] 0.5677419
```

(e) Recall that a consistent system of equations satisfies $\mathbf{AA}^- \mathbf{y} = \mathbf{y}$. Is our system consistent?

```
>A** gA**y
```

R returns

```
      [,1]
[1,]  6
[2,]  4
```

(f) Use R to compute $\mathbf{I} - \mathbf{A}^{-1}\mathbf{A}$

```
> I <- matrix(c(1,0,0,0,1,0,0,0,1),nrow=3)
> I - gA% ** A
R returns
```

```
      [,1]      [,2]      [,3]
[1,]  0.725806 -0.09677419 -0.43548387
[2,] -0.0967742  0.01290323  0.05806452
[3,] -0.4354839  0.05806452  0.26129032
```

(g) Recall that for a consistent system, all solutions can be written as $\mathbf{x} = \mathbf{A}^{-1}\mathbf{y} + (\mathbf{I} - \mathbf{A}^{-1}\mathbf{A})\mathbf{c}$, where \mathbf{c} is any vector of constants. What is the family of solutions for this equation?

$$x_1 = 0.387 + 0.726c_1 - 0.097c_2 - 0.435c_3$$

$$x_2 = 0.348 - 0.097c_1 + 0.013c_2 + 0.058c_3$$

$$x_3 = 0.568 - 0.435c_1 + 0.058c_2 + 0.261c_3$$