

Genetics 320, Problem Set Ten

Due Wednesday 10 December 2003 at 11 am

1 (1 point): Recall that the number of mutations follows a Poisson distribution. For a population of size $N = 10^5$ and a mutation rate of $u = 10^{-6}$,

(a) What is the probability that a random individual is a heterozygote?

Here $4Nu = 4 \cdot 10^5 \cdot 10^{-6} = 0.4$, hence

$$H = \frac{4Nu}{1 + 4Nu} = \frac{0.4}{1.4} = 0.286$$

(b) Assuming that the common ancestor to two random sequences occurs $2N$ generations ago, what is the probability that two randomly-chosen sequences differ by one mutation? By two mutations? By five mutations?

If the time back to the common ancestor is $2N$ generations, then the expected number of new mutations is $2 \cdot (2N) \cdot u = 0.4$, and the probability of k mutations follows a Poisson distribution with parameter 0.4. Hence,

$$\text{Prob}(1 \text{ mutation}) = (0.4)^1 e^{-0.4} / 1! = 0.268.$$

$$\text{Prob}(2 \text{ mutations}) = (0.4)^2 e^{-0.4} / 2! = 0.054.$$

$$\text{Prob}(5 \text{ mutations}) = (0.4)^5 e^{-0.4} / 5! = 0.000057$$

2 (2 points): In an infinite population, what happens for each of the following fitnesses

- (a) $W_{AA} = 1, W_{Aa} = 1.001, W_{aa} = 1.002$ *a fixed*
- (b) $W_{AA} = 0.95, W_{Aa} = 0.94, W_{aa} = 0.85$ *A fixed*
- (c) $W_{AA} = 1, W_{Aa} = 1, W_{aa} = 0.75$ *A fixed*
- (d) $W_{AA} = 1, W_{Aa} = 0.75, W_{aa} = 0.75$ *A fixed*
- (e) $W_{AA} = W_{Aa} = W_{aa}$ *Hardy-Weinberg. Alleles frequencies unchanged.*

3 (2 points): Suppose we sample 1000 individuals and the number of offspring they leave. Focusing on a specific diallelic locus, we find the following:

Genotypes:	AA	Aa	aa
Numbers observed	160	480	360
Average number of offspring:	10.5	12.0	12.5

(a) Setting $W_{aa} = 1$, what are the relative fitnesses of the three genotypes?

$$W_{AA} = 10.5/12.5 = 0.84, W_{Aa} = 12.0/12.5 = 0.96, \text{ and } W_{aa} = 1.$$

(b) What is the change in the frequency of allele A after one generation of selection?

Freq(A after selection) = Freq(AA after selection) + (1/2) Freq(Aa after selection). Here, initial freq(AA) = 0.16, freq(Aa) = 0.48, freq(aa) = 0.36, giving the initial frequency of A as $0.16 + (1/2) 0.48 = 0.4$.

$$\bar{W} = W_{AA} \cdot 0.16 + W_{Aa} \cdot 0.48 + W_{aa} \cdot 0.36 = 0.84 \cdot 0.16 + 0.96 \cdot 0.48 + 1 \cdot 0.36 = 0.9552$$

$$\text{Freq}(AA \text{ after selection}) = 0.16 \cdot (W_{AA} / \bar{W}) = 0.16 \cdot (0.84 / 0.9552) = 0.14$$

$$\text{Freq}(Aa \text{ after selection}) = 0.48 \cdot (W_{Aa} / \bar{W}) = 0.48 \cdot (0.96 / 0.9552) = 0.482$$

$$\text{Hence, Freq}(A \text{ after selection}) = 0.14 + (1/2) \cdot 0.482 = 0.381$$

4 (2 points): Suppose the fitnesses at a locus are $W_{AA} = 1 - s$, $W_{Aa} = 1$, and $W_{aa} = 1 - t$. For $\text{freq}(A) = t/(s + t)$, show that

(a) $W_A = W_a = \bar{W}$.

$$W_A = pW_{AA} + (1 - p)W_{Aa} = \left(\frac{t}{s + t}\right) \cdot (1 - s) + \left(\frac{s}{s + t}\right) \cdot 1 = \frac{t - ts + s}{s + t}$$

$$W_a = pW_{Aa} + (1 - p)W_{aa} = \left(\frac{t}{s + t}\right) \cdot 1 + \left(\frac{s}{s + t}\right) \cdot (1 - t) = \frac{t - ts + s}{s + t}$$

Hence, $W_A = W_a$. Since $\bar{W} = pW_A + (1 - p)W_a$, it follows that $\bar{W} = W_A = W_a$.

(b) Showing that the change in $\text{freq}(A) =$ the change in $\text{freq}(a) = 0$.

Recalling that $\Delta p_A = p_A(W_A - \bar{W})/\bar{W}$, since $W_A = \bar{W}$, it immediately follows that $\Delta p_A = 0$. The same argument holds for Δp_a .

5 (2 points): Consider a locus with five alleles. Two populations are considered, with the following allele frequencies and marginal fitnesses:

	Population one				
Allele:	A_1	A_2	A_3	A_4	A_5
Frequency:	0.1	0.2	0.5	0.1	0.1
Marginal fitness:	3.5	1.2	4.0	3.3	6.2

	Population Two				
Allele:	A_1	A_2	A_3	A_4	A_5
Frequency:	0.2	0.1	0.1	0.3	0.3
Marginal fitness:	1.5	2.5	2.0	4.5	2.4

Compute the change in the frequency of allele A_3 in both populations.

$$\bar{W} = \sum_{i=1}^5 W_{A_i} \text{Freq}(A_i). \text{ Hence, for populations one and two,}$$

$$\bar{W}(\text{pop 1}) = 0.1 \cdot 3.5 + 0.2 \cdot 1.2 + 0.5 \cdot 4.0 + 0.1 \cdot 3.3 + 0.1 \cdot 6.2 = 3.54$$

$$\bar{W}(\text{pop 2}) = 0.2 \cdot 1.5 + 0.1 \cdot 2.5 + 0.1 \cdot 2.0 + 0.3 \cdot 4.5 + 0.3 \cdot 2.4 = 2.82$$

$$\text{Hence, in population one, } \Delta p_3 = p_3(W_3 - \bar{W})/\bar{W} = 0.5 \cdot (4.0 - 3.54)/3.54 = 0.065$$

$$\text{In population two, } \Delta p_3 = p_3(W_3 - \bar{W})/\bar{W} = 0.1 \cdot (2.0 - 2.82)/2.82 = -0.029$$

6 (1 point): Individuals homozygous for the sickle-cell allele (ss) die before reproducing. However, normal/sickle heterozygotes (Ss) have greater fitness than normal homozygotes (SS) due to increased resistance to malaria. If the frequency of allele s in equilibrium populations is 0.2, what is the fitness advantage of Ss relative to SS individuals?

Under balancing selection, if the fitnesses of $W_{ss} : W_{sS} : W_{SS}$ are $0 : 1 : 1 - t$, then the frequency of allele s at equilibrium is $t/(t + 1)$. Thus, $0.2 = t/(t + 1)$ or $0.2 \cdot (t + 1) = t$ or $0.2 = 0.8t$ or $t = 0.2/0.8 = 0.25$. Hence, $W_{sS}/W_{SS} = 1/(1 - t) = 1/0.75 = 1.33$.