Genetics 320, Problem Set Four Solutions

1 (1 point): If $\Pr(A, B) = 0.15$, $\Pr(A) = 0.2$ and $\Pr(B) = 0.4$,

(a) What is the conditional probability of B given A?

Pr(A,B)/P(A) = 0.15/0.2 = 0.75

(b) What is the conditional probability of A given B?

Pr(A,B)/P(B) = 0.15/0.4 = 0.375

2 (2 points): Suppose the frequency of a disease in the population is 0.005 and the relative risk for this disease (among sibs) is 35.

(a) What is the probability that you have this disease given that your sib does?

$RR \cdot K = 35 \cdot 0.005 = 0.175$

(b) What is the expected frequency of two-sib families where both sibs have the disease?

$RR \cdot K^2 = 35 \cdot 0.005^2 = 0.000875$

(c) Suppose we look at 1,000 randomly-chosen two-sib families. What is the probability that we will observe no families where both sibs have the disease?

Pr(none) $= (1 - 0.000875)^{1000} = 0.41670$.

3 (1 point): Consider five offspring from a pair of Aa parents. What is the probability that no offspring are aa? That all five are aa?

Pr(0) = $(1 - p)^5 = (3/4)^5 = 0.237$, ***Pr(5) =*** $(p)^5 = (1/4)^5 = 0.00097$

4 (1 point): The **Binomial distribution** appears frequently in genetics and gives the probability of k successes in n trials, given that the success probability per trial is p, as

$$\Pr(k \text{ successes in } n \text{ trials}) = \frac{n!}{(n-k)!k!}\, p^k (1-p)^{n-k}$$

Here $n!$ is n factorial where $n! = n \cdot (n-1) \cdots 1$ (this function is found on most hand-held calculators.) Using the Binomial distribution, if the chance of getting a boy is 0.53, what is the probability that a family of five (children) has three boys?

Pr(3) = $5!/[3!(5-3)!)]p^3(1-p)^{(5-3)} = \frac{120}{6 \cdot 2}(0.53)^3(0.47)^2 = 0.3289$

5 (1 point): Consider the offspring from a pair of $AaBbCc$ parents, where all three loci are unlinked.

(a) What is the probability of getting an $aabbcc$ offspring? (Hint: unlinked implies that the resulting genotypes at each locus are independent of the other loci.)

Pr(aabbcc) = Pr(aa)· Pr(bb)· Pr(cc) = $(1/4) \cdot (1/4) \cdot (1/4) = 0.0156$

(b) Compute the probabilities that we see at least one $aabbcc$ offspring among the first 10, 50, 100, and 250 offspring (respectively).

The chance of success on each trial is 0.0156, and the probability that we see at least one offspring in T ***tries is just 1-Pr(see none in*** T***) tries =*** $1 - (1 - p)^T$.
For T = 10, $1 - (1 - 0.0156)^{10} = 0.1457$***, likewise***
For T = 50, 0.5450, For T = 100, 0.7930, and For T = 250, 0.9805

6 (1 point): Consider the offspring of a pair of parents who are both $AaBb$, where the loci are unlinked. Let A– denote either AA or Aa, and likewise B– denote either BB or Bb.

(a) What is the probability of an AAB– offspring?

Pr(AAB–) = Pr(AA)*Pr(B–) = Pr(AA)*[Pr(BB) + Pr(Bb)] = (1/4)*(1/4+1/2) = (1/4)*(3/4) = 3/16

(b) Of an A–B– offspring?

Pr(A–B–) = Pr(A–)*Pr(B–) = (3/4)*(3/4) = 9/16

7 (1 point): Another very common distribution see in genetics is the **Poisson distribution**, which gives the probability of observing k successes in our sample given that the expected number of successes of λ as

$$\Pr(k \text{ successes }) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Note that $0! = 1$ (by definition), and recall that $\lambda^0 = 1$. Suppose we are looking at a large cell culture and we expect 2.5 mutants (on average) in the culture. Using the Poisson distribution,

(a) what is the probability that we see no mutants?

$$Pr(0) = \frac{2.5^0 e^{-2.5}}{0!} = e^{-2.5} = 0.08$$

(b) what is the probability that we see more than two mutants (i.e., three or more mutants)?

$$Pr(\geq 3) = 1 - [\,\Pr(0) + \Pr(1) + \Pr(2)\,] = 1 - e^{-2.5}\left(1 + \frac{2.5}{1} + \frac{2.5^2}{2}\right) = 0.456$$

8 (2 points): There are genetic conditions that result in families having children of only one sex (typically females). Suppose the population prevalence of the all-female disorder is 0.01.

(a) What is the chance that a family that has had 6 girls will have a boy as their 7th child? (Hint, use conditional probabilities to account for the two possible conditions of (i) the family being normal and (ii) the family having this genetic predisposition to all females)

Pr(boy) = Pr(boy | normal family)*Pr(normal family) + Pr(boy | sex-biased condition) * Pr(sex-biased condition) = (1/2)*(1-0.01) + 0*0.01 = 0.495

(b) Suppose that the 7th child also turns out to be a girl. One can use **Bayes' theorem** to compute the probability that this family indeed has this genetic predisposition to all girls (as opposed to this simply occurring by chance). Bayes' theorem is as follows: suppose there are n possible outcomes $(b_1, b_2, \cdots, b_n)$ of a random variable that we cannot observe. Given the observed outcome of a correlated variable A, what is the probability of b_j? Bayes' theorem states that

$$\Pr(b_j \mid A) = \frac{\Pr(b_j)\Pr(A \mid b_j)}{\sum_{i=1}^{n} \Pr(b_i)\ \Pr(A \mid b_i)}$$

To apply this to our problem, let A = event of 7 girls, b_1 = normal family, and b_2 = genetic predisposition to all girls.

Let sbc = sex-biased (all female) condition (here Pr(female) = 1) and nf = normal family (here Pr(female) = 1/2). Hence,
Pr(7 girls | nf) = $(1/2)^7 = 0.0078$***, Pr(7 girls | sbc) = 1, Pr(nf) = 1-0.01 = 0.99, Pr(sbc) = 0.01.***
Putting these together

$$\textit{Pr(sbc} \mid \textit{7 girls}) = \frac{\textit{Pr(7 girls} \mid \textit{sbc}) \cdot \textit{Pr(sbc)}}{\textit{Pr(7 girls} \mid \textit{sbc}) \cdot \textit{Pr(sbc)} + \textit{Pr(7 girls} \mid \textit{nf}) \cdot \textit{Pr(nf)}}$$

$$= \frac{1 \cdot 0.01}{1 \cdot 0.01 + 0.0078 \cdot 0.99} = 0.5643$$