Lecture 17

Multi-Trait Selection

Reference

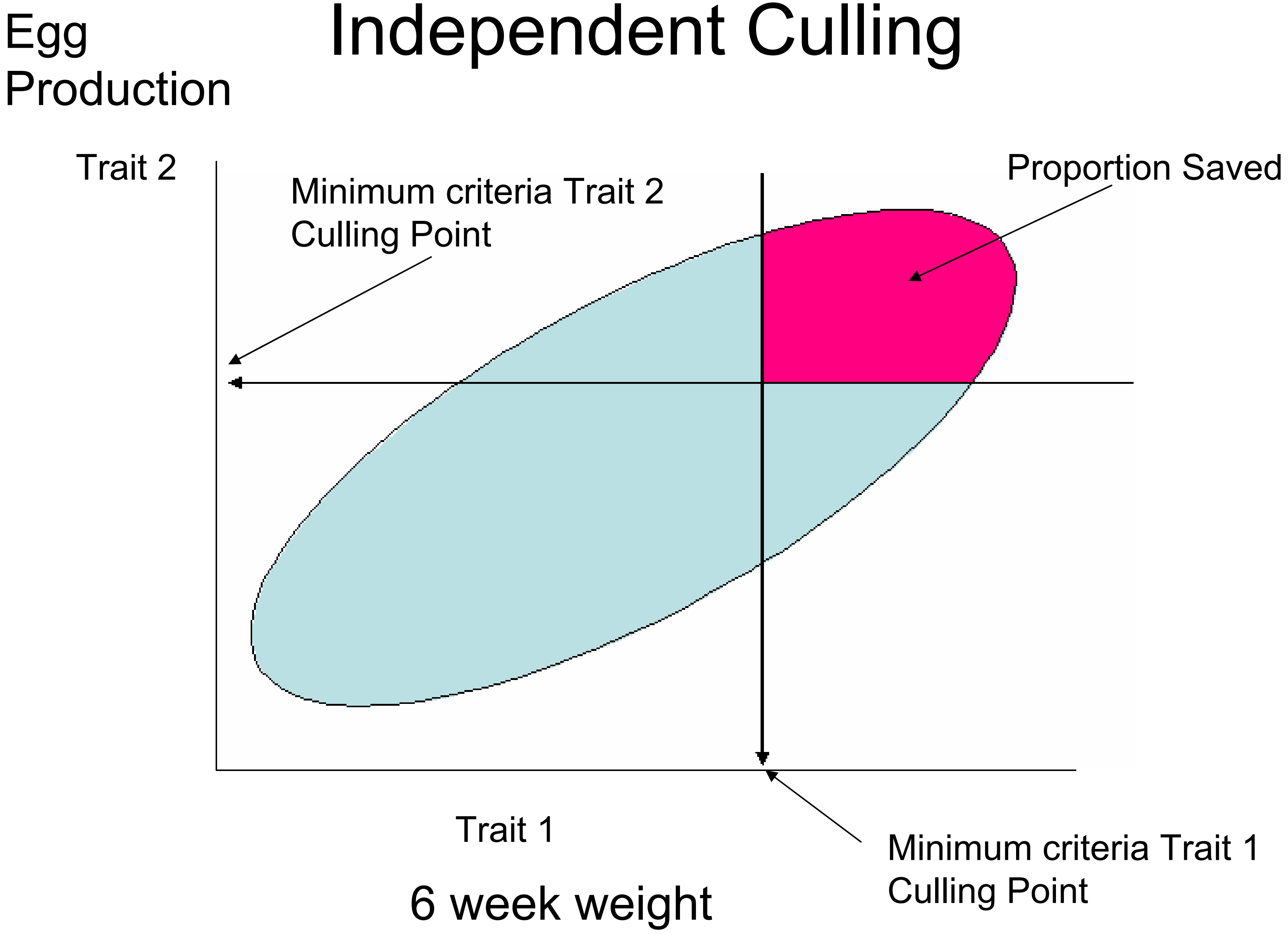
Walsh and Lynch Ch 23, 24

Xu and Muir (1992)

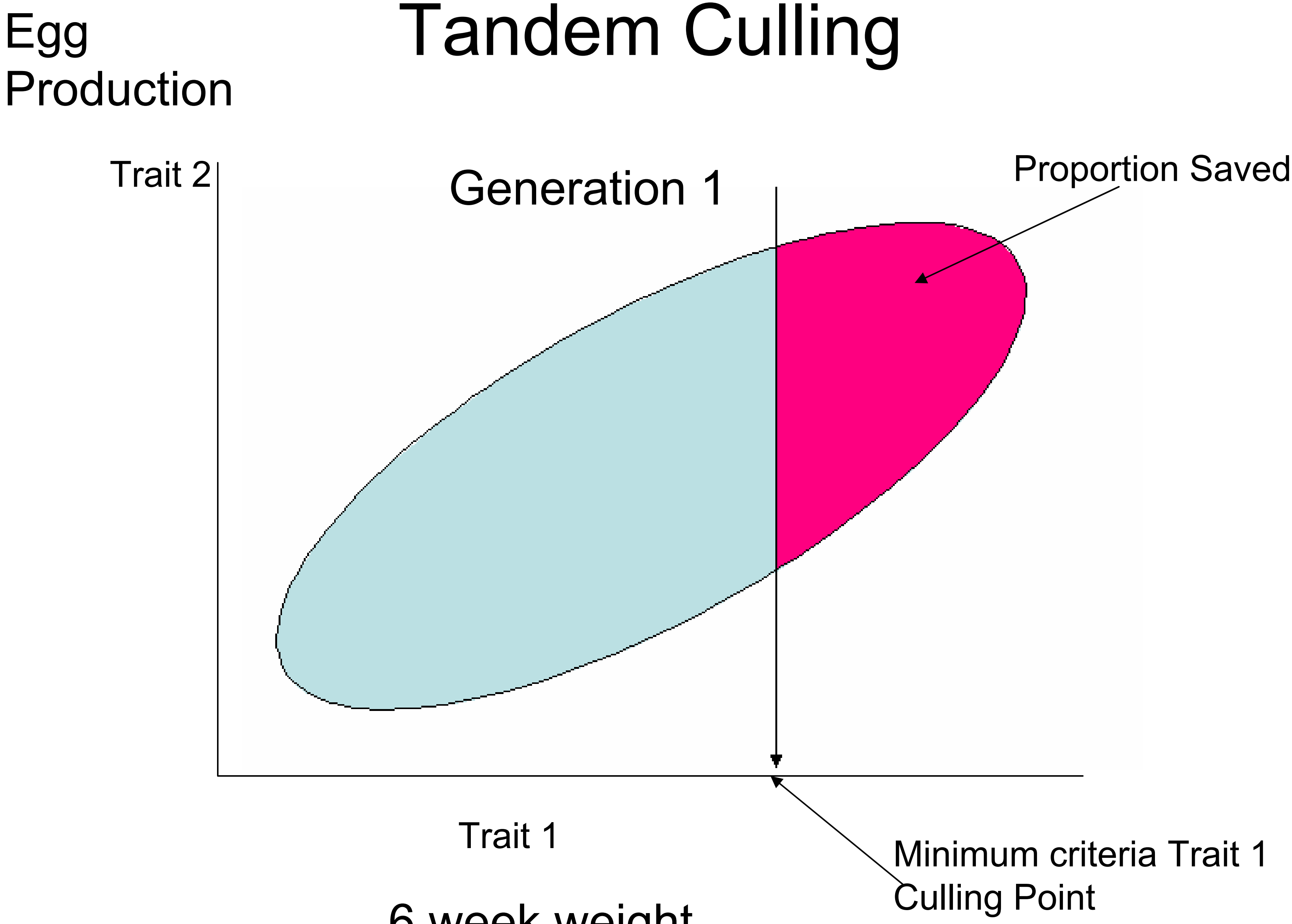
Three Methods

- Independent Culling Levels
- Tandem
- Index
- Combination
  - Independent Culling and Index
    - Multi-stage Selection Index Updating
    - Xu and Muir (1992)

Independent Culling

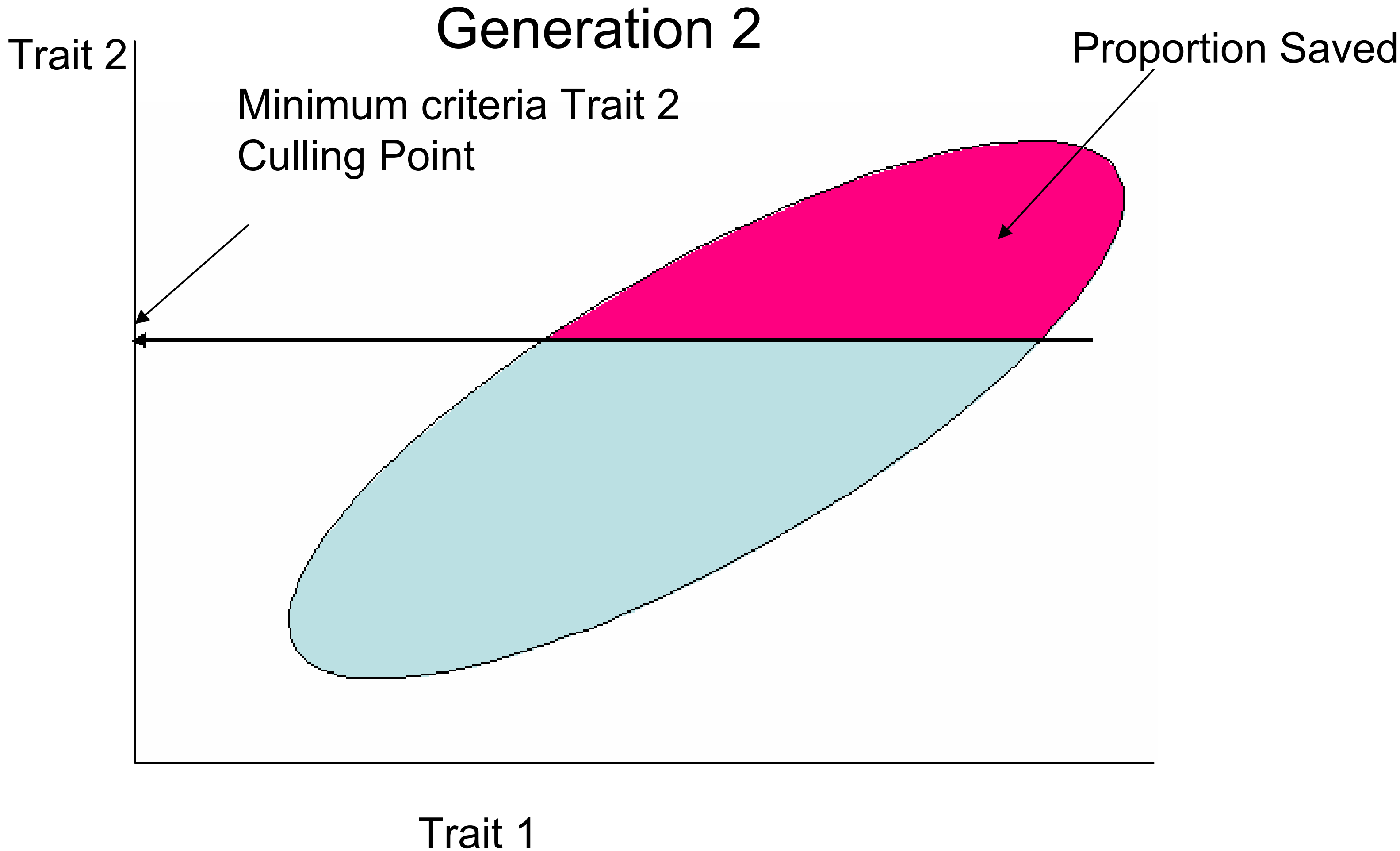


Tandem Culling



Tandem Culling

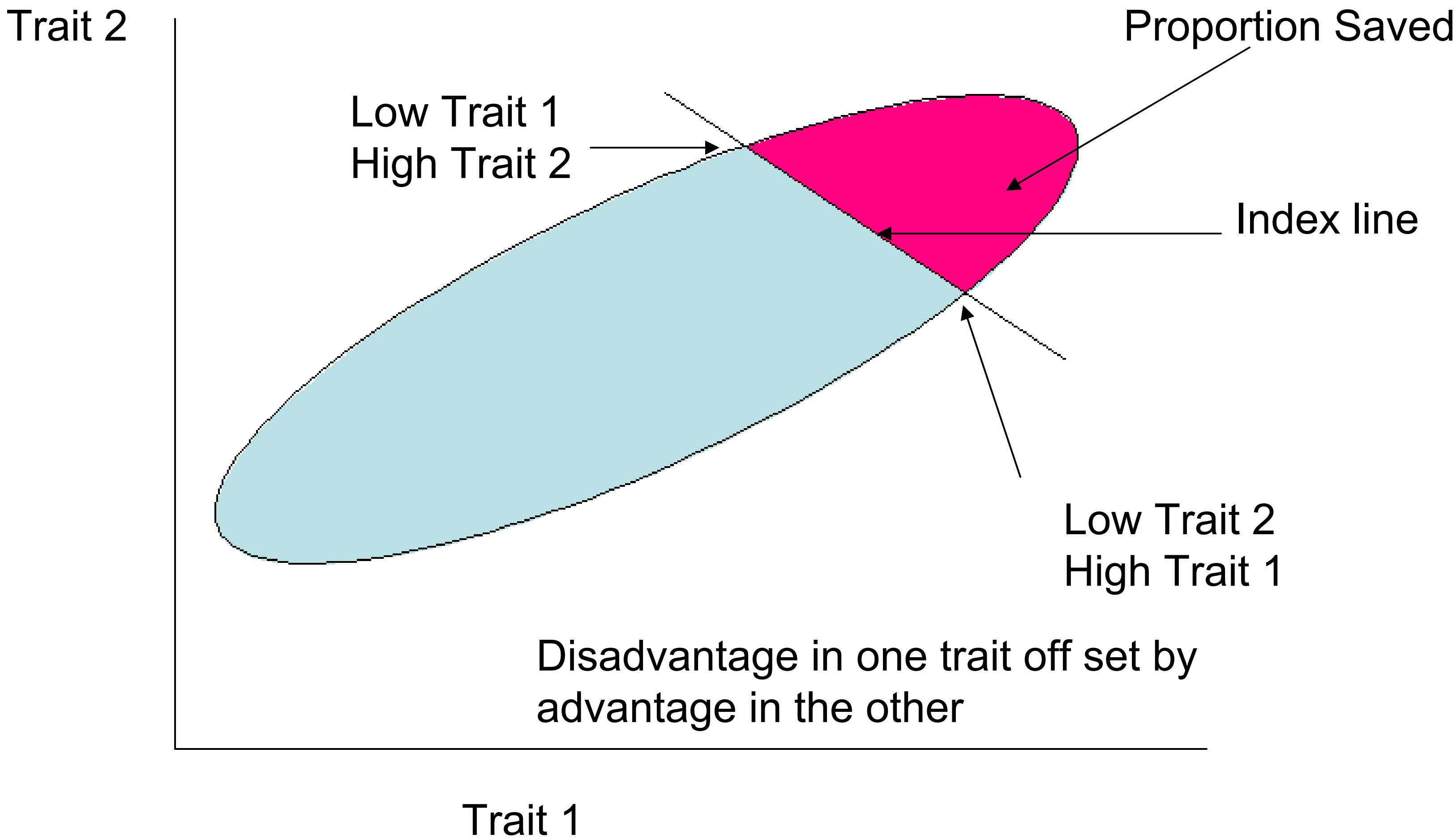
Egg Production



6 week weight

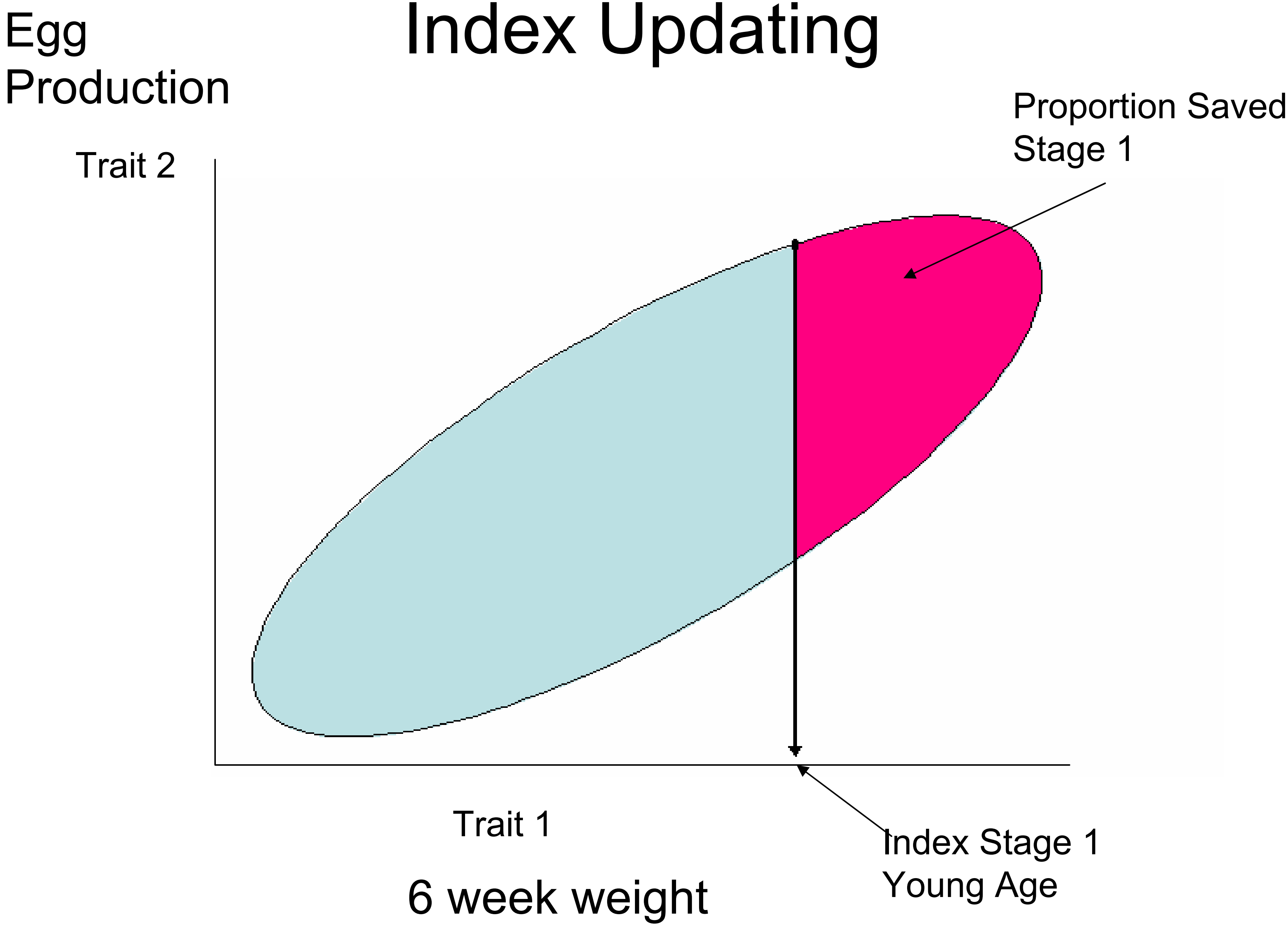
Index Culling

Egg Production



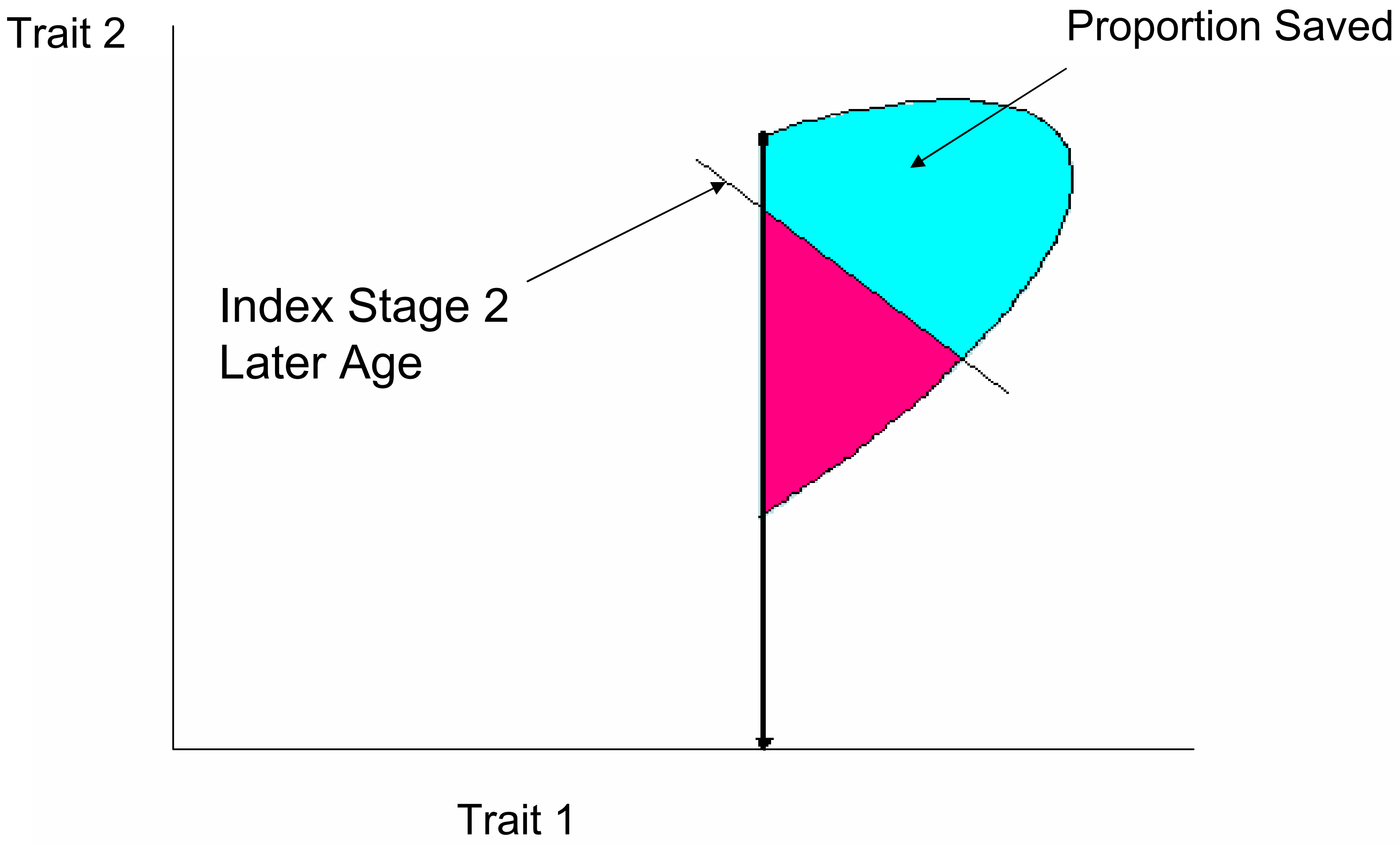
6 week weight

Index Updating



Index Updating

Egg Production



6 week weight

Theoretical Comparisons

- Same total Number of Individuals Measured
  - Genetic Gain
    - Index Selection > Index Updating > Independent Culling > Tandem Selection
  - So Why Use Anything Else But Index?
    - **Economics**
    - **Selection Intensity**

Economics

Traits Measured At Different Life Stages

- 6 Week Weight and Egg production in Broiler Dam Lines
- Index Selection
  - Must Keep ALL animals until ALL traits are Measured
  - Cull in one stage
- Independent Culling and Index Updating
  - Cull Animals As Traits Become Available
  - Independent Culling has NO advantages over Index Updating

Economics

Traits Differ Greatly in Costs to Measure

- Genotypic and Phenotypic Measures
  - Genotyping More Expensive than Phenotype
    - Meat Quality
    - Measure Phenotype First
    - Measure Genotype on Remainder
  - Phenotype More Expensive than Genotype
    - Annual Egg Number
    - Measure Genotype First
    - Measure Phenotype on Remainder
- Use Index Updating

Selection Intensity

- Limitations of Facilities may be different for different ages
  - Total Number of Individuals Measured Can be **Greater** For Independent Culling or Index Updating Than Index
  - Selection Intensity may be Greater for multi-stage Selection
  - Depending on Selection Intensity
    - Index Updating > Independent Culling> Index Selection

Example

- Limited Facilities
  - Index Updating or Independent Culling
    - 2 Stage
    - Measure **10,000** chicks Stage 1
    - **1000** Hens Stage 2
  - Index
    - Single Stage
    - Measure **1,000** chicks
    - Same **1000** Hens Later
- Index Selection is More Accurate
- Index Updating or Independent Culling May Have Greater Selection Intensity

Prediction of Response to Selection: Single Trait

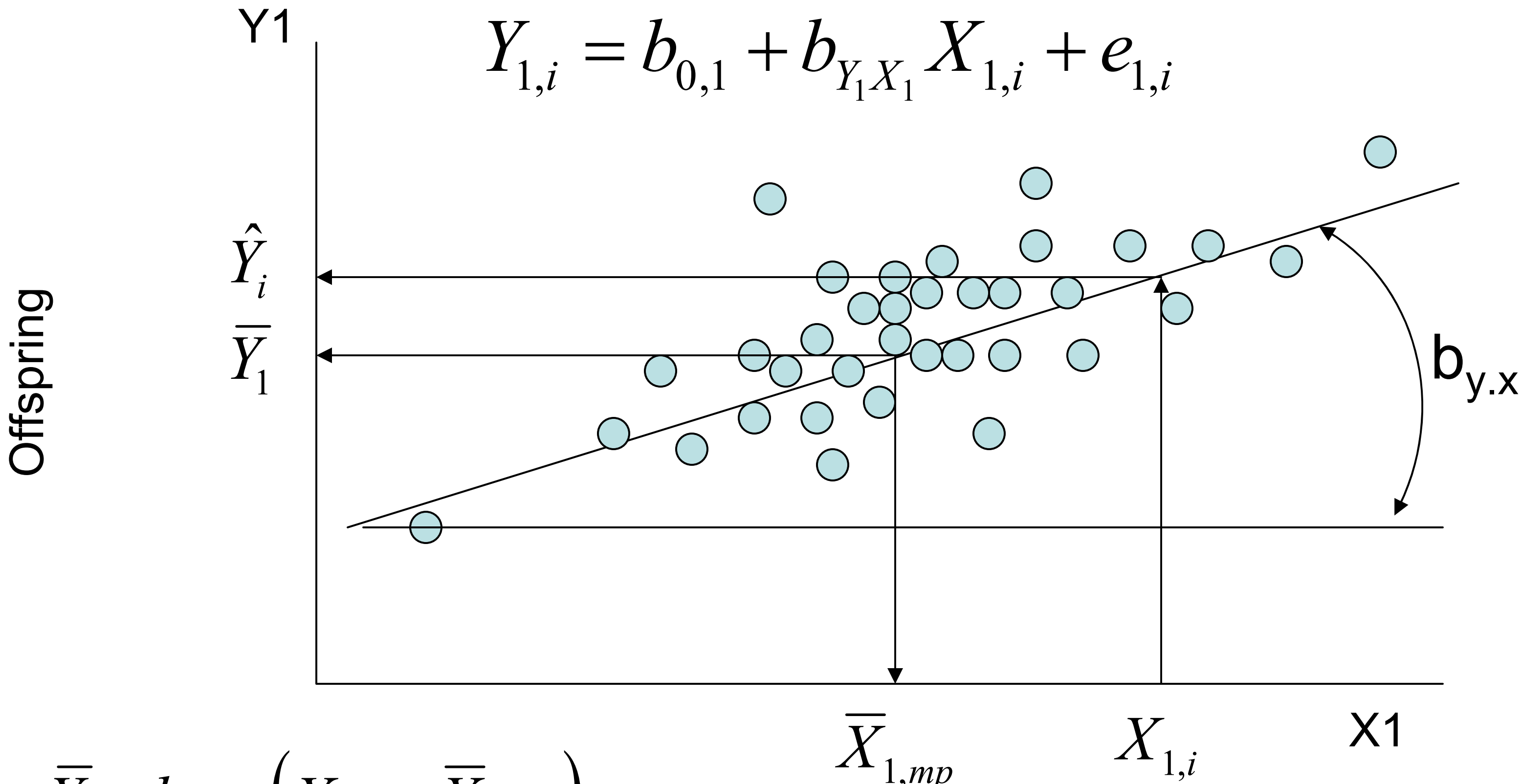
Prediction is a process of going from a known to an unknown

- Know parental values, want to predict offspring performance
- Linear Regression: Method of Choice
  - Simple
  - Accurate within bounds
  - Taylor Expansion

Prediction

Prior Information needed to develop calibration curve

X and Y same trait Measured in Different Generations



$$\hat{Y}_{1,i} = \overline{Y}_1 + b_{Y_1.X_1}\left(X_{1,i} - \overline{X}_{1,mp}\right)$$

Parental Averages

Selection on Trait 1, Prediction of Response of traits 1

$$\hat{Y}_{1,i} = \overline{Y}_1 + b_{Y_1.X_1}\left(X_{1,i} - \overline{X}_{1,mp}\right)$$

$$\hat{Y}_{1,i} = b_0 + b_{Y_1.X_1} X_{1,i} \qquad b_0 = \overline{Y}_1 - b_{Y_1.X_1}\overline{X}_{1,mp}$$

$$b_{Y_1.X_1} = \frac{\sum_i \left(Y_{1,i} - \overline{Y}_1\right)\left(X_{1,i} - \overline{X}_1\right)}{\sum_i \left(X_{1,i} - \overline{X}_1\right)^2} \qquad b_{Y_1.X_1} = \frac{\sum_i \left(Y_{1,i} - \overline{Y}_1\right)\left(X_{1,i} - \overline{X}_1\right)/(n-1)}{\sum_i \left(X_{1,i} - \overline{X}_1\right)^2/(n-1)}$$

$$b_{Y_1.X_1} = \frac{Cov(Y_1, X_1)}{V(X_1)} = \frac{\hat{\sigma}_{X_1 Y_1}}{\hat{\sigma}^2_{X_1}} = \frac{\frac{1}{2}\hat{\sigma}^2_{A_1}}{\frac{1}{2}\hat{\sigma}^2_{P_1}} = h_1^2$$

$$\hat{Y}_{1,i} = \overline{Y}_1 + h_1^2 SD_1 \qquad \hat{Y}_{1,i} = \overline{X}_1 + h_1^2 SD_1$$

$$\hat{Y}_{1,i} = \overline{X}_1 + h_1^2\left(i_1 \sigma_{P_1}\right)$$

In Matrix Terms: Normal Equations

$$\begin{bmatrix} n & \sum X_{1,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_{Y_1 . X_1} \end{bmatrix} = \begin{bmatrix} \sum Y_{1,i} \\ \sum X_{1,i} Y_{1,i} \end{bmatrix}$$

Worked As a Deviation From the Mean

$$\left(\hat{Y}_{1i} - \overline{Y}_1\right) = b_{Y_1 . X_1}\left(X_{1,i} - \overline{X}_{1,mp}\right)$$

Normal Equations

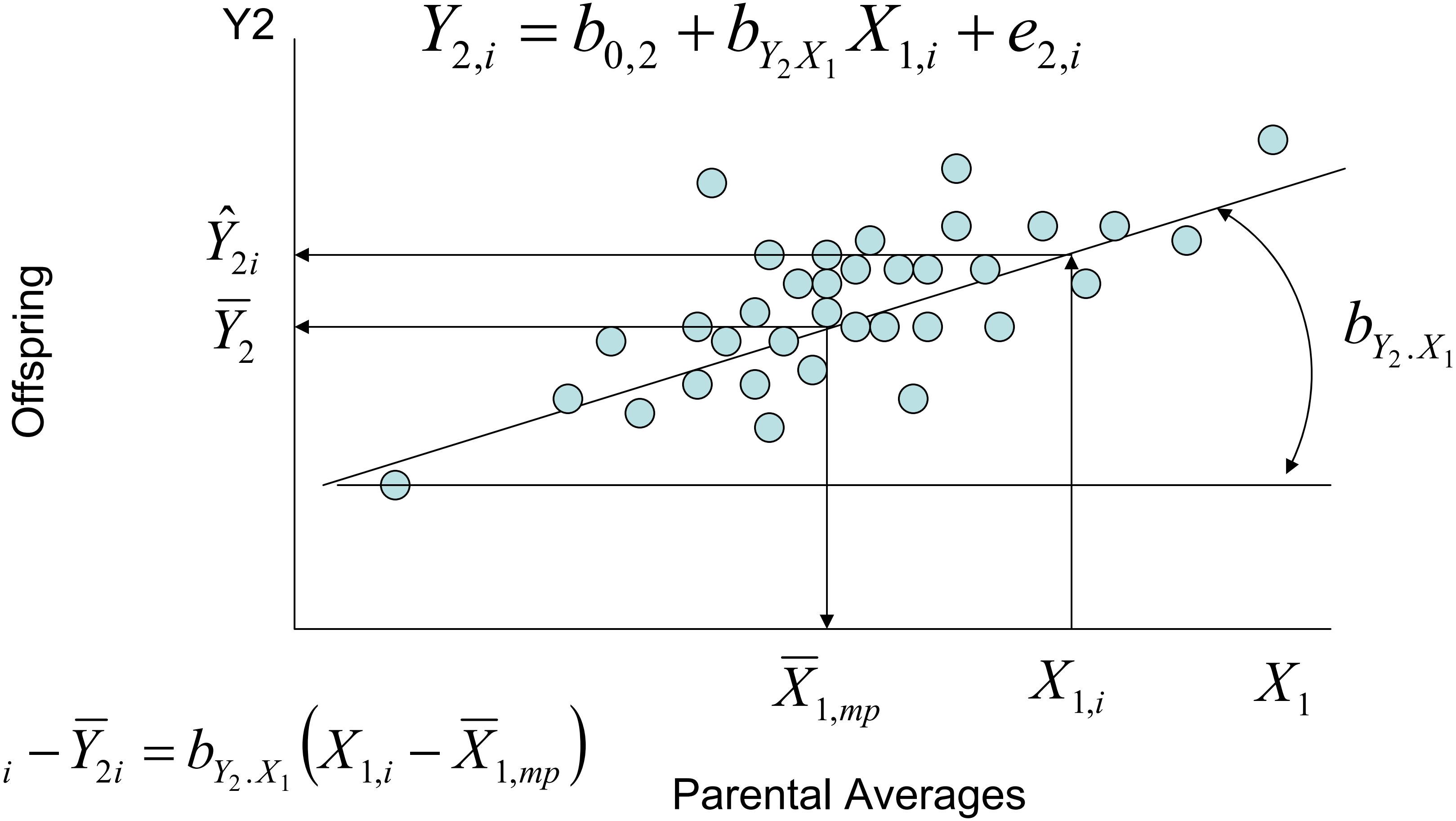
$$\left[\sigma^2_{P_1}\right]\left[b_{Y_1 . X_1}\right] = \left[\sigma^2_{G_1}\right]$$

Prediction of a Second Trait

Prior Information needed to develop calibration curve

X and Y **Different** traits Measured in Different Generations

$$Y_{2,i} = b_{0,2} + b_{Y_2 X_1} X_{1,i} + e_{2,i}$$



$$\hat{Y}_{2i} - \overline{Y}_{2i} = b_{Y_2 . X_1}\left(X_{1,i} - \overline{X}_{1,mp}\right)$$

The Mean of trait 2 in the offspring can be predicted from the deviation of trait 1 in the parents

$$\hat{Y}_{2i} = \overline{Y}_{2i} + b_{Y_2.X_1}\left(X_{1,i} - \overline{X}_{1,mp}\right) \qquad b_{Y_2.X_1} = \frac{\sum_i \left(Y_{2,i} - \overline{Y}_2\right)\left(X_{1,i} - \overline{X}_1\right)}{\sum_i \left(X_{1,i} - \overline{X}_1\right)^2}$$

$$b_{Y_2.X_1} = \frac{Cov(Y_2, X_1)}{V(X_1)} = \frac{\hat{\sigma}_{x1,y2}}{\hat{\sigma}^2_{x1}} = \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}^2_{P_1}}$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}^2_{P_1}}\left(X_{1,i} - \overline{X}_{1,mp}\right)$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}^2_{P_1}} i_1 \hat{\sigma}_{P_1} \qquad \hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{P_1}} i_1$$

The Mean of trait 2 in the offspring can be predicted from the deviation of trait 1 in the parents

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{P_1}} i_1$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{P_1}\hat{\sigma}_{P_2}} i_1 \hat{\sigma}_{P_2}$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{P_1}\hat{\sigma}_{P_2}} \left(\frac{\hat{\sigma}_{A_1}}{\hat{\sigma}_{A_1}}\right)\left(\frac{\hat{\sigma}_{A_2}}{\hat{\sigma}_{A_2}}\right) i_1 \hat{\sigma}_{P_2}$$

Selection on Trait 1, Predicting Response of Trait 2

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{P_1}\hat{\sigma}_{P_2}}\left(\frac{\hat{\sigma}_{A_1}}{\hat{\sigma}_{A_1}}\right)\left(\frac{\hat{\sigma}_{A_2}}{\hat{\sigma}_{A_2}}\right) i_1 \hat{\sigma}_{P_2}$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + \frac{\hat{\sigma}_{A_{1,2}}}{\hat{\sigma}_{A_1}\hat{\sigma}_{A_2}}\left(\frac{\hat{\sigma}_{A_1}}{\hat{\sigma}_{P_1}}\right)\left(\frac{\hat{\sigma}_{A_2}}{\hat{\sigma}_{P_2}}\right) i_1 \hat{\sigma}_{P_2}$$

$$\hat{Y}_{2,i} = \overline{Y}_2 + r_{G_{1,2}} h_1 h_2 i_1 \hat{\sigma}_{P_2}$$

Worked As a Deviation From the Mean

$$\left(\hat{Y}_{2i} - \bar{Y}_2\right) = b_{Y_2.X_1}\left(X_{1i} - \bar{X}_{1,mp}\right)$$

$$\left[\sum\left(X_{1,i} - \bar{X}_{1,mp}\right)^2\right]\left[b_{Y_2.X_1}\right] = \left[\sum\left(X_{1,i} - \bar{X}_{1,mp}\right)\left(Y_{2,i} - \bar{Y}_2\right)\right]$$

$$\left[\frac{\sum\left(X_{1,i} - \bar{X}_{1,mp}\right)^2}{n-1}\right]\left[b_{Y_2.X_1}\right] = \left[\frac{\sum\left(X_{1,i} - \bar{X}_{1,mp}\right)\left(Y_{2,i} - \bar{Y}_2\right)}{n-1}\right]$$

$$\left[\sigma^2_{P_1}\right]\left[b_{Y_2.X_1}\right] = \left[\sigma_{G_{1,2}}\right]$$

Select on Traits 1 and 2
Predict Response of Trait 1

$$Y_{1,i} = b_{1.0} + b_{1.1} X_{1,i} + b_{1.2} X_{2,i} + e_{1,i}$$

$$\left(Y_{1,i} - \overline{Y}_1\right) = b_{1.1}\left(X_{1,i} - \overline{X}_1\right) + b_{1.2}\left(X_{2,i} - \overline{X}_2\right) + e_{1,i}$$

Normal Equations

$$\begin{bmatrix} \sum\left(X_{1,i} - \overline{X}_1\right)^2 & \sum\left(X_{1,i} - \overline{X}_1\right)\left(X_{2,i} - \overline{X}_2\right) \\ \sum\left(X_{1,i} - \overline{X}_1\right)\left(X_{2,i} - \overline{X}_2\right) & \sum\left(X_{2,i} - \overline{X}_2\right)^2 \end{bmatrix} \begin{bmatrix} b_{1.1} \\ b_{1.2} \end{bmatrix} = \begin{bmatrix} \sum\left(X_{1,i} - \overline{X}_1\right)\left(Y_{1,i} - \overline{Y}_1\right) \\ \sum\left(X_{2,i} - \overline{X}_2\right)\left(Y_{1,i} - \overline{Y}_1\right) \end{bmatrix}$$

Divide Both sides by n-1

$$\begin{bmatrix} \sigma^2_{P_1} & \sigma_{P_{1,2}} \\ \sigma_{P_{1,2}} & \sigma^2_{P_2} \end{bmatrix} \begin{bmatrix} b_{1.1} \\ b_{1.2} \end{bmatrix} = \begin{bmatrix} \sigma^2_{A_1} \\ \sigma_{A_{1,2}} \end{bmatrix}$$

Select on Traits 1 and 2
Predict Response of Trait 2

$$Y_{2,i} = b_{2.0} + b_{2.1} X_{1,i} + b_{2.2} X_{2,i} + e_{2,i}$$

$$\left(Y_{2,i} - \overline{Y}_2\right) = b_{2.1}\left(X_{1,i} - \overline{X}_1\right) + b_{2.2}\left(X_{2,i} - \overline{X}_2\right) + e_{2,i}$$

Normal Equations

$$\begin{bmatrix} \sum\left(X_{1,i} - \overline{X}_1\right)^2 & \sum\left(X_{1,i} - \overline{X}_1\right)\left(X_{2,i} - \overline{X}_2\right) \\ \sum\left(X_{1,i} - \overline{X}_1\right)\left(X_{2,i} - \overline{X}_2\right) & \sum\left(X_{2,i} - \overline{X}_2\right)^2 \end{bmatrix} \begin{bmatrix} b_{2.1} \\ b_{2.2} \end{bmatrix} = \begin{bmatrix} \sum\left(X_{1,i} - \overline{X}_1\right)\left(Y_{2,i} - \overline{Y}_2\right) \\ \sum\left(X_{2,i} - \overline{X}_2\right)\left(Y_{2,i} - \overline{Y}_2\right) \end{bmatrix}$$

Divide Both sides by n-1

$$\begin{bmatrix} \sigma^2_{P_1} & \sigma_{P_{1,2}} \\ \sigma_{P_{1,2}} & \sigma^2_{P_2} \end{bmatrix} \begin{bmatrix} b_{2.1} \\ b_{2.2} \end{bmatrix} = \begin{bmatrix} \sigma_{A_{1,2}} \\ \sigma^2_{A_2} \end{bmatrix}$$

Select on Traits 1 and 2
Predict Response of Traits 1 and 2

$$\left(Y_{1,i}-\overline{Y}_1\right)=b_{1.1}\left(X_{1,i}-\overline{X}_1\right)+b_{1.2}\left(X_{2,i}-\overline{X}_2\right)+e_{1,i}$$

$$\left(Y_{2,i}-\overline{Y}_2\right)=b_{2.1}\left(X_{1,i}-\overline{X}_1\right)+b_{2.2}\left(X_{2,i}-\overline{X}_2\right)+e_{2,i}$$

$$\begin{bmatrix}\Delta Y_1\\ \Delta Y_2\end{bmatrix}=\begin{bmatrix}b_{1.1} & b_{1.2}\\ b_{2.1} & b_{2.2}\end{bmatrix}\begin{bmatrix}\Delta X_1\\ \Delta X_2\end{bmatrix}+\begin{bmatrix}e_1\\ e_2\end{bmatrix}$$

$$\mathbf{\Delta Y}=\mathbf{B}\left(\mathbf{\Delta X}\right)+\mathbf{E}$$

Normal Equations

$$\begin{bmatrix}\sum\left(X_{1,i}-\overline{X}_1\right)^2 & \sum\left(X_{1,i}-\overline{X}_1\right)\left(X_{2,i}-\overline{X}_2\right)\\ \sum\left(X_{1,i}-\overline{X}_1\right)\left(X_{2,i}-\overline{X}_2\right) & \sum\left(X_{2,i}-\overline{X}_2\right)^2\end{bmatrix}\begin{bmatrix}b_{1.1} & b_{2.1}\\ b_{1.2} & b_{2.2}\end{bmatrix}$$

$$=\begin{bmatrix}\sum\left(X_{1,i}-\overline{X}_1\right)\left(Y_{1,i}-\overline{Y}_1\right) & \sum\left(X_{2,i}-\overline{X}_2\right)\left(Y_{2,i}-\overline{Y}_2\right)\\ \sum\left(X_{2,i}-\overline{X}_2\right)\left(Y_{1,i}-\overline{Y}_1\right) & \sum\left(X_{2,i}-\overline{X}_2\right)\left(Y_{2,i}-\overline{Y}_2\right)\end{bmatrix}$$

Select on Traits 1 and 2
Predict Response of Traits 1 and 2

$$\begin{bmatrix} \sum\left(X_{1,i}-\bar{X}_1\right)^2 & \sum\left(X_{1,i}-\bar{X}_1\right)\left(X_{2,i}-\bar{X}_2\right) \\ \sum\left(X_{1,i}-\bar{X}_1\right)\left(X_{2,i}-\bar{X}_2\right) & \sum\left(X_{2,i}-\bar{X}_2\right)^2 \end{bmatrix}\begin{bmatrix} b_{1.1} & b_{2.1} \\ b_{1.2} & b_{2.2} \end{bmatrix}$$

$$=\begin{bmatrix} \sum\left(X_{1,i}-\bar{X}_1\right)\left(Y_{1,i}-\bar{Y}_1\right) & \sum\left(X_{2,i}-\bar{X}_2\right)\left(Y_{2,i}-\bar{Y}_2\right) \\ \sum\left(X_{2,i}-\bar{X}_2\right)\left(Y_{1,i}-\bar{Y}_1\right) & \sum\left(X_{2,i}-\bar{X}_2\right)\left(Y_{2,i}-\bar{Y}_2\right) \end{bmatrix}$$

Divide Both sides by n-1

$$\begin{bmatrix} \sigma^2_{P_1} & \sigma_{P_{1,2}} \\ \sigma_{P_{1,2}} & \sigma^2_{P_2} \end{bmatrix}\begin{bmatrix} b_{1.1} & b_{2.1} \\ b_{1.2} & b_{2.2} \end{bmatrix}=\begin{bmatrix} \sigma^2_{A_1} & \sigma_{A_{1,2}} \\ \sigma_{A_{1,2}} & \sigma^2_{A_2} \end{bmatrix}$$

$$\mathbf{PB}=\mathbf{G}$$

In General

$$\begin{bmatrix} \Delta Y_1 \\ \Delta Y_2 \end{bmatrix} = \begin{bmatrix} b_{1.1} & b_{1.2} \\ b_{2.1} & b_{2.2} \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$\Delta \mathbf{Y} = \mathbf{B}(\Delta \mathbf{X}) + \mathbf{E}$$

$$\mathbf{PB} = \mathbf{G} \qquad \hat{\mathbf{B}} = \mathbf{P}^{-1}\mathbf{G}$$

$$\Delta \hat{\mathbf{Y}} = \hat{\mathbf{B}}(\Delta \mathbf{X})$$

$$\Delta \hat{\mathbf{Y}} = \mathbf{P}^{-1}\mathbf{G}(\Delta \mathbf{X})$$

$$\Delta \hat{\mathbf{Y}} = \begin{bmatrix} \Delta \hat{\mathbf{Y}}_1 \\ \Delta \hat{\mathbf{Y}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{\Delta \hat{G}_1} \\ \mathbf{\Delta \hat{G}_2} \end{bmatrix}$$

Genetic Gain In trait 1

Genetic Gain In trait 2

Determining Which Traits to Select for And How Much

- Economics
  - Which Traits
    - will produce the greatest profits?
    - will Respond Most Rapidly?
    - Are Economical to Measure?
    - Are some traits which are easy to measure but with low or no economic value predictors of other more difficult or expensive to measure?

Economic Weights

a_i Economic return that will result for each unit increase in the i^{th} trait

ΔH_j Aggregate economic breeding value of the j^{th} individual

$$\Delta\hat{\mathbf{G}} = \mathbf{P}^{-1}\mathbf{G}\left(\Delta\mathbf{X}\right)$$

For a given individual, assume the value of the mate is the population mean, then predict the effect of selecting that individual on the offspring

$$\Delta H_j = \sum_i a_i \Delta\hat{G}_{ij}$$

Economic Breeding Value of the jth Individual

Simply Chose those individuals with the highest EBV

Breeding Program

- Method of simply choosing individuals with highest EBV's is easy to implement but does not address the mode difficult theoretical question of which traits to measure
- Need a more theoretical predictive approach

Traits Measured and Selection Objective

- Selection Criterion
  - Traits Measured and selected upon
- Selection Objective
  - Traits Improved
- Criterion and Objective may Not be the Same
  - Back Fat
    - Selection Criterion
    - easy to measure
  - Feed Efficiency
    - Selection Objective
    - expensive to measure

Optimal Selection Index

Let Z be the Selection Criterion
An index with arbitrary weights on each of m traits

$$Z = w_1 X_1 + w_2 X_2 \cdots + w_m X_m$$

Let Q be the Selection Objective
The aggregate Economic Gain in p traits From Selecting On Index Z

$$Q = a_1 \Delta G_1 + a_2 \Delta G_2 \cdots + a_p \Delta G_p$$

Problem: Find Z such that genetic gain for Q is maximum

Find Z such that genetic gain for Q is maximum

$$Z = w_1 X_1 + w_2 X_2 \cdots + w_m X_m$$

$$Z = \mathbf{w}'\mathbf{X}$$

Think of Z at a trait which has several sub-components

$$Q = a_1 \Delta G_1 + a_2 \Delta G_2 \cdots + a_p \Delta G_p$$

$$Q = \mathbf{a}'\boldsymbol{\Delta}\mathbf{G}$$

Think of Q at another trait which has several sub-components

Want to Predict Change in Q from Selection on Z

Usual Single Trait Selection Problem

Predict Change in Q from Selection on Z

$$\hat{Q} = \overline{Q} + b_{Q.Z}\left(i_1 \sigma_Z\right)$$

$$\hat{Q} = \overline{Q} + \frac{Cov(Q,Z)}{\sqrt{V(Z)}} i_1$$

Where $$Q = \mathbf{a}'\boldsymbol{\Delta}\mathbf{G} \qquad Z = \mathbf{w}'\mathbf{X}$$

$$\hat{Q} = \overline{Q} + \frac{Cov(\mathbf{a}'\boldsymbol{\Delta}\mathbf{G}, \mathbf{w}'\mathbf{X})}{\sqrt{V(\mathbf{w}'\mathbf{X})}} i_1$$

$$\hat{Q} = \overline{Q} + \frac{\mathbf{w}' Cov(\boldsymbol{\Delta}\mathbf{G}, \mathbf{X})\mathbf{a}}{\sqrt{\mathbf{w}'V(\mathbf{X})\mathbf{w}}} i_1$$

$$\hat{Q} = \overline{Q} + \frac{\mathbf{w}' Cov(\Delta\mathbf{G}, \mathbf{X})\mathbf{a}}{\sqrt{\mathbf{w}' V(\mathbf{X})\mathbf{w}}} i_1$$

$Cov(\Delta\mathbf{G}, \mathbf{X})$

$\mathbf{G}$

Genetic Variance Covariance Matrix between all traits measured and all traits in the objective

$V(\mathbf{X})$

$\mathbf{P}$

Phenotypic Variance Covariance Matrix between all traits measured

$$COV(Q, Z) = \mathbf{w}'\mathbf{Ga}$$

$$V(Z) = \mathbf{w}'\mathbf{Pw}$$

We want the regression to be maximum

$$\hat{Q} = \overline{Q} + \underbrace{\frac{\mathbf{w}'\mathbf{G}\mathbf{a}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}}}_{\text{Maximize}} i_1$$

Let $$R = \frac{\mathbf{w}'\mathbf{G}\mathbf{a}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}}$$

$$\ln(R) = \ln(\mathbf{w}'\mathbf{G}\mathbf{a}) - \ln\left(\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}\right)$$

Maximize Response in Aggregate Economic Gain

$$\ln(R) = \ln\left(\mathbf{w}'\mathbf{Ga}\right) - \tfrac{1}{2}\ln\left(\mathbf{w}'\mathbf{Pw}\right)$$

Set $$\frac{\delta \ln(R)}{\delta \mathbf{w}} = 0$$

Solve for $\mathbf{w}$

$$\frac{\delta \ln(R)}{\delta \mathbf{w}} = \left(\frac{1}{\mathbf{w}'\mathbf{Ga}}\right)\mathbf{Ga} - \tfrac{1}{2}\left(\frac{1}{\mathbf{w}'\mathbf{Pw}}\right)\left(2\mathbf{Pw}\right) = 0$$

$$\mathbf{Ga} = k\mathbf{Pw}$$

$$\mathbf{w} = \mathbf{P}^{-1}\mathbf{Ga}$$ Optimal Weights

Predicting Genetic Gain in Each Trait

For any arbitrary set of weights w (including optimal)

$$Z = w_1 X_1 + w_2 X_2 \cdots + w_m X_m$$

$$Z = \mathbf{w}'\mathbf{X}$$

$$\Delta\hat{\mathbf{G}} = \mathbf{P}^{-1}\mathbf{G}(\Delta\mathbf{X})$$

$$\Delta\hat{\mathbf{G}} = \frac{\mathbf{w}'\mathbf{G}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}} i_Z$$

Choice of Traits

Now You can play around with different traits in the index and examine theoretical impacts

Example Broilers

Table 1. Traits measured.

Trait	WHEN MEASURED	MEAN	PHENOTYPIC STD. DEV.	COST ($) TO MEASURE
5 week weight (WT5)	DAY 35	1238g	104g	1.15
6 week weight (WT6)	DAY 42	1591g	141g	1.15
Feed efficiency (FE=100xGAIN/FEED)	DAYS 35-42	48.7%	3.58%	12.55
VLDL	(DAY 43)	.217	.057	2.15
Egg production (PR=part record)	(WKS 24-36)	54 eggs	8 eggs	45.84

Note Hi Cost to Measure Feed Efficiency and Egg Production

Economic Weights

Table 2. Traits of economic importnace.

Trait	MEAN	PHENOTYPIC STD. DEV.	Economic Weight ($)
6 week weight (WT6)	1591g	141g	.022/g
Feed efficiency (FE=100xGAIN/FEED)	48.7%	3.58%	.296/%
Fat	48g	9g	0
Egg production (PR=part record)	54 eggs	8 eggs	.50/egg
Egg production (RR=residual record)	122 eggs	18 eggs	.50/egg

Genetic Parameters

Table 3. Genetic, phenotypic correlations, and heritabilities, respectively above, below and on diagonal.

TRAIT	WT5	WT6	FE	VLDL	FAT	PR	RR
WT5	.30	.87	.50	-.20	.42	-.05	-.07
WT6	.91	.35	.60	-.22	.38	-.09	-.10
FE	.40	.50	.25	-.20	-.13	.10	.10
VLDL	-.10	-.12	.05	.50	.55	0.0	0.0
FAT	.54	.66	-.10	.45	.60	-.10	-.10
PR	.08	.10	.15	0.0	-.10	.15	.65
RR	.08	.10	.15	0.0	-.10	.75	.15

Optimal Index Weight and Eggs

	$wt6$	PR	RR
$wt6$	h_1^2	$r_{G_{1,2}}$	$r_{G_{1,3}}$
PR	$r_{P_{1,2}}$	h_2^2	$r_{G_{2,3}}$
RR	$r_{P_{1,3}}$	$r_{P_{2,3}}$	h_3^2

	$wt6$	PR	RR
$wt6$	.35	−.09	−.10
PR	.10	.15	.65
RR	.10	.75	.15

$$\mathbf{G} = \begin{bmatrix} 6958 & -23.3 & -58.4 \\ -23.3 & 9.6 & 14.1 \\ -58.4 & 14.1 & 48.6 \end{bmatrix} \qquad \mathbf{P} = \begin{bmatrix} 19881 & 112.8 & 253.8 \\ 112.8 & 64 & 108 \\ 253.8 & 108 & 48.6 \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} .022 \\ .5 \\ .5 \end{bmatrix}$$

Optimal Weights

Traits measured = wt6 and PR

Find optimal weights on the two traits for total economic gain and gain in each trait

RR missing value but can predict

$$\mathbf{G} = \begin{bmatrix} 6958 & -23.3 & -58.4 \\ -23.3 & 9.6 & 14.1 \\ & & \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 19881 & 112.8 & \\ 112.8 & 64 & \\ & & \end{bmatrix}$$

$$\mathbf{w} = \mathbf{P}^{-1}\mathbf{G}\mathbf{a}$$

Optimal Weights

$$\hat{Q} - \overline{Q} = \frac{\mathbf{w}'\mathbf{G}\mathbf{a}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}} i_1$$

Genetic Gain in Economic Value

$$\Delta\hat{\mathbf{G}} = \frac{\mathbf{w}'\mathbf{G}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}} i_Z$$

Genetic Gain in each trait

SAS PROGRAM

```
proc iml;
start main;

P={19881 112.8 ,
  112.8  64   };

G={6958 -23.3 -58.4,
  -23.3 9.6  14.1};

a={.022,
  .5,
  .5};

w=inv(P)*G*a;
cov=w`*G*a;
VZ=w`*p*w;
SZ=sqrt(vz);
DH=cov*inv(SZ);
DG=w`*G*inv(SZ);
print w DH DG;
finish main;
run;
quit;
```

SAS OUTPUT

W DH DG

0.0046867 1.5622743 18.354496 0.9678917 1.349059
0.1688866

Optimal Weights for Each Trait

Expected Improvement in Economic Return

Expected Change in each trait

Arbitrary Weights

Traits measured = wt6 and PR

Use Given Weight for the two traits, Find expected change in each trait and economic value

$$\mathbf{G} = \begin{bmatrix} 6958 & -23.3 & -58.4 \\ -23.3 & 9.6 & 14.1 \\ & & \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 19881 & 112.8 & \\ 112.8 & 64 & \\ & & \end{bmatrix}$$

$$\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Set Weights

$$\Delta\hat{\mathbf{G}} = \frac{\mathbf{w}'\mathbf{G}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}} i_Z$$

Genetic Gain in each trait

$$\hat{Q} - \overline{Q} = \frac{\mathbf{w}'\mathbf{G}\mathbf{a}}{\sqrt{\mathbf{w}'\mathbf{P}\mathbf{w}}} i_1$$

Genetic Gain in Economic Value

SAS PROGRAM

```
proc iml;
start main;

P={19881 112.8 ,
  112.8  64   };

G={6958 -23.3 -58.4,
  -23.3 9.6  14.1};

a={.022,
  .5,
  .5};

W={1,
     1};

cov=w`*G*a;
VZ=w`*p*w;
SZ=sqrt(vz);
DH=cov*inv(SZ);
DG=w`*G*inv(SZ);
print w DH DG;
finish main;
run;
quit;
```

SAS Output

W DH DG

1 0.8700224 48.827925 -0.096463 -0.311921
1

Set Weights for Each Trait

Expected Change in each trait

Note Change in Economic Value Much Less than Optimal

Lab Problem 1

- If one measure Wt6, VLDL(fat), and PR
  - A. What are the optimal weights
  - B. What is the expected change in aggregate economic gain considering all traits of economic importance
- If one measured all traits except RR
  - A. What are the optimal Weight
  - B. What is the expected change in aggregate economic gain considering all traits of economic importance
- Considering the costs of both programs which one do you think would be more profitable?

```
*****************************************************
*                 INDUPDAT                    *
*     Selection Index Updating for Maximum        *
*    Economic Return Using the EM Type Algorithm    *
*                     By                     *
*                 Shizhong Xu                   *
*              Rutgers University                *
*                July 4, 1992                   *
*****************************************************
;
OPTION LINESIZE=79;
PROC IML;
* P IS AN NXN PHENOTYPIC VARIANCE-COVARIANCE MATRIX;
P={
10816.000  13344.240 148.928  -.593  66.56,
13344.240  19881     252.39   -.932  112.8,
148.928    252.39    12.816   .010   4.296,
-.593      -.932     .010     .003   0,
66.56      112.8     4.296    0      64};
* G IS AN NXK GENETIC COVARIANCE MATRIX;
G={
3244.8  4133.961 50.982 -.459  166.787 -8.825  -27.798,
4133.961 6958.35 89.59  -.726  220.981 -23.261 -58.153,
50.982   89.59   3.204  -.014  -1.622  .555    1.248,
-.459    -.726   -.014  .002   .155    0       0,
-8.825   -23.261  .555   0     -2.16    9.6     14.04};
* W IS A KX1 VECTOR OF ECONOMIC WEIGHTS;
W={0, .0011, .0148, 0, 0, .5, .5};
* TOTAL PROPORTION SELECTED ;
PT=1/15;
* N IS AN 1XM VECTOR OF NUMBER OF TRAITS WITHIN STAGES;
N={2 2 1};
*N={5};
* MAXIMIZE GAIN OR RATIO OR PROFIT?;
MAX='PROFIT';
* VCOST IS AN NX1 VECTOR OF COST PER TRAIT;
VCOST={.15, .150 ,.57, 1.15,5.4};
KAPA=50;
BETA=1;

*PROGRAM STARTS;

m=ncol(n);
c=n`;
ii=n[1]; c[1]=sum(vcost[1:ii]);
do jj=2 to m;
ii=ii+n[jj];
c[jj]=sum(vcost[ii-n[jj]+1:ii]);
end;
PI=3.1415926;
X=PROBIT(1-PT); T=EXP(-X*X/2)/SQRT(2*PI);I=T/PT;
PRINT 'VARIANCE COVARIANCE MATRICES AND ECONOMIC WEIGHTS';
PRINT P; PRINT G; PRINT W c;
PRINT 'TOTAL PROPORTION AND SELECTION INTENSITY';
PRINT PT I;
B=P**-1*G*W;
DG=G`*B*I/SQRT(B`*P*B); DH=W`*DG;
PRINT 'ONE STAGE INDEX SELECTION';
E=PT*BETA*KAPA;
COST=C[+];
PROFIT=E*DH-COST;
RATIO=DH/COST;
START MOD;
IF MAX='RATIO' THEN GOTO R;
IF MAX='PROFIT' THEN GOTO P;
PRINT B DG DH;GOTO G;
R:PRINT B DG DH COST RATIO;
GOTO G;
P:PRINT B DG DH COST PROFIT;

G:I1=I;
B=P[1:N[1],1:N[1]]**-1*G[1:N[1],]*W; I=N[1];
DO J=2 TO M;
I=I+N[J]  ;
QII=P[1:I,1:I]; QIJ=P[1:I-N[J],1:I]; AI=G[1:I,];
BI=(I(I)-QII**-1*QIJ`*B*(B`*QIJ*QII**-1*QIJ`*B)**-1*B`*QIJ)
   *QII**-1*AI*W;
B=(B//J(N[J],J-1,0))||BI;
END; VI=B`*P*B;
B=B*SQRT(DIAG(VI))**-1;
A=W`*G`*B;
PRINT 'MULTI-STAGE SELECTION';
PRINT B;
```

```
START SC(K,Q,C,SC); SC=0; CQ=1;
DO I=1 TO K;
SC=SC+C[I]*CQ;
CQ=CQ*Q[I];
END;
FINISH SC;

start dz(q,dz);
u=probit(j(nrow(q),1,1)-q);
dz=exp(-.5*u##2)/sqrt(2*3.1415926)/q;
finish dz;

q=j(m,1,pt##(1/m));
call sc(m,q,c,cost);
call dz(q,dz);
IF MAX='RATIO' THEN GOTO R1;
IF MAX='PROFIT' THEN GOTO P1;
profit=a*dz;GOTO G1;
R1:profit=a*dz/cost;
GOTO G1;
P1:profit=e*a*dz-cost;

g1:iter=0;
a:iter=iter+1;
do i=1 to m-1;
do j=i+1 to m;
r=q[i]*q[j];
qi=q[i];qj=q[j];
call sc(m,q,c,cost);
call dz(q,dz);
IF MAX='RATIO' THEN GOTO R2;
IF MAX='PROFIT' THEN GOTO P2;
prf0=a*dz;GOTO G2;
R2:prf0=a*dz/cost;
GOTO G2;
P2:prf0=e*a*dz-cost;
g2:vb=.9999;
va=r+.0001;
b:d=(vb-va)/5;
if d<.0001 then goto c;
do f=va to vb by d;
q[i]=f; q[j]=r/q[i];
call sc(m,q,c,cost);
call dz(q,dz);
IF MAX='RATIO' THEN GOTO R3;
IF MAX='PROFIT' THEN GOTO P3;
prf1=a*dz;GOTO G3;
R3:prf1=a*dz/cost;
GOTO G3;
P3:prf1=e*a*dz-cost;

g3:if prf1>prf0 then do;
prf0=prf1; qi=q[i]; qj=q[j];
end;end;
if (qi-d)>va then va=qi-d;
if (qi+d)<vb then vb=qi+d;
goto b;
c:q[i]=qi; q[j]=qj;
end; end;
if ((prf1-profit)<.000001 | iter>50) then goto f;
profit=prf1;
goto a;
f: profit=prf1;
ratio=profit;
dg=g`*b*dz;
dh=w`*dg;
u=probit(j(m,1,1)-q);
print 'Final Results';
IF MAX='RATIO' THEN GOTO R4;
IF MAX='PROFIT' THEN GOTO P4;
print iter u q dz dg dh;goto g4;
R4:print iter u q dz dg dh cost ratio;
GOTO G4;
P4:print iter u q dz dg dh cost profit;
g4:finish mod;run mod;
print 'B   - transformation matrix from y to z';
print 'ITER- number of iterations            ';
print 'U   - truncation points               ';
print 'Q   - proportions selected            ';
print 'DZ  - selection differentials         ';
print 'DG  - genetic gains                   ';
print 'DH  - total genetic gain              ';
print 'COST- cost of measurement             ';
print 'RATIO  - gain to cost ratio           ';
print 'PROFIT - profit                       ';
```

Index Updating

```
 ONE STAGE INDEX SELECTION

    B          DG            DH        COST     PROFIT
  0.0066673 -11.11706  3.063772 7.42    2.7925732
 -0.008675 -31.08097
  0.1146971 0.0764045
 -2.094819  -0.00314
  0.1850733 -2.106555
  2.4353458
```

MULTI-STAGE SELECTION

MULTI-STAGE SELECTION

B

0.0150606	0.0036846	0.0002105
-0.015502	-0.006832	-0.000383
0	0.3278797	-0.037348
0	-4.189784	0.0472402
0	0	0.12649

DG	DH	COST	PROFIT
-16.81431	2.4499631	2.4854298	5.6811138
-50.37834			
-0.182759			
-0.001648			
-2.514702			
1.9198849			
3.0962833			

Lab Problem 2

- Examine Alternative Multi-Stage selection programs to optimize profits