

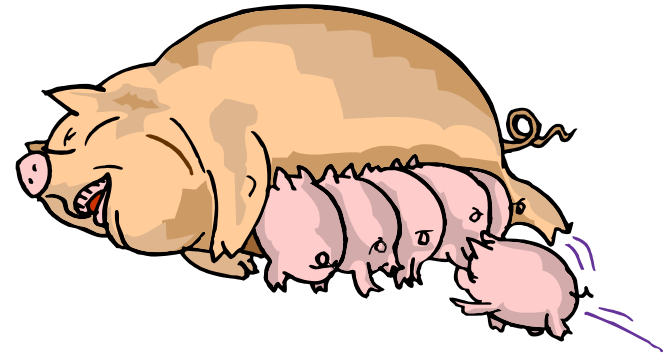
# Lecture 14

## Maternal Effects-Inherited

Reference: Lynch and Walsh Ch 23  
Schaeffer, LR Linear Models and Computing  
Strategies in Animal Breeding

# Maternal Effects

- Maternal
  - Genetic (Nuclear DNA)
    - Mendelian Segregation
    - Inherited maternal effects
      - Milking ability
      - Mothering ability
  - Genetic (Cytoplasmic DNA)
    - Transmitted along maternal lines
  - Permanent Environmental
    - Non inherited maternal effects
      - Mastitis
      - Other maternal infection
      - Maternal Injuries (Damaged teats)





# Maternal Genetic Effects

$$y = \mathbf{Xb} + \mathbf{Z}_1\mathbf{d} + \mathbf{Z}_2\mathbf{m} + \mathbf{e}$$

Direct effect

Maternal Genetic

$$V \begin{bmatrix} \mathbf{d} \\ \mathbf{m} \\ \mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A}\sigma_d^2 & \mathbf{A}\sigma_{d,m} & 0 \\ \mathbf{A}\sigma_{d,m} & \mathbf{A}\sigma_m^2 & 0 \\ 0 & 0 & \mathbf{I}\sigma_e^2 \end{bmatrix}$$

No environmental correlations

There is a genetic correlation between the animals direct and maternal genetic effect

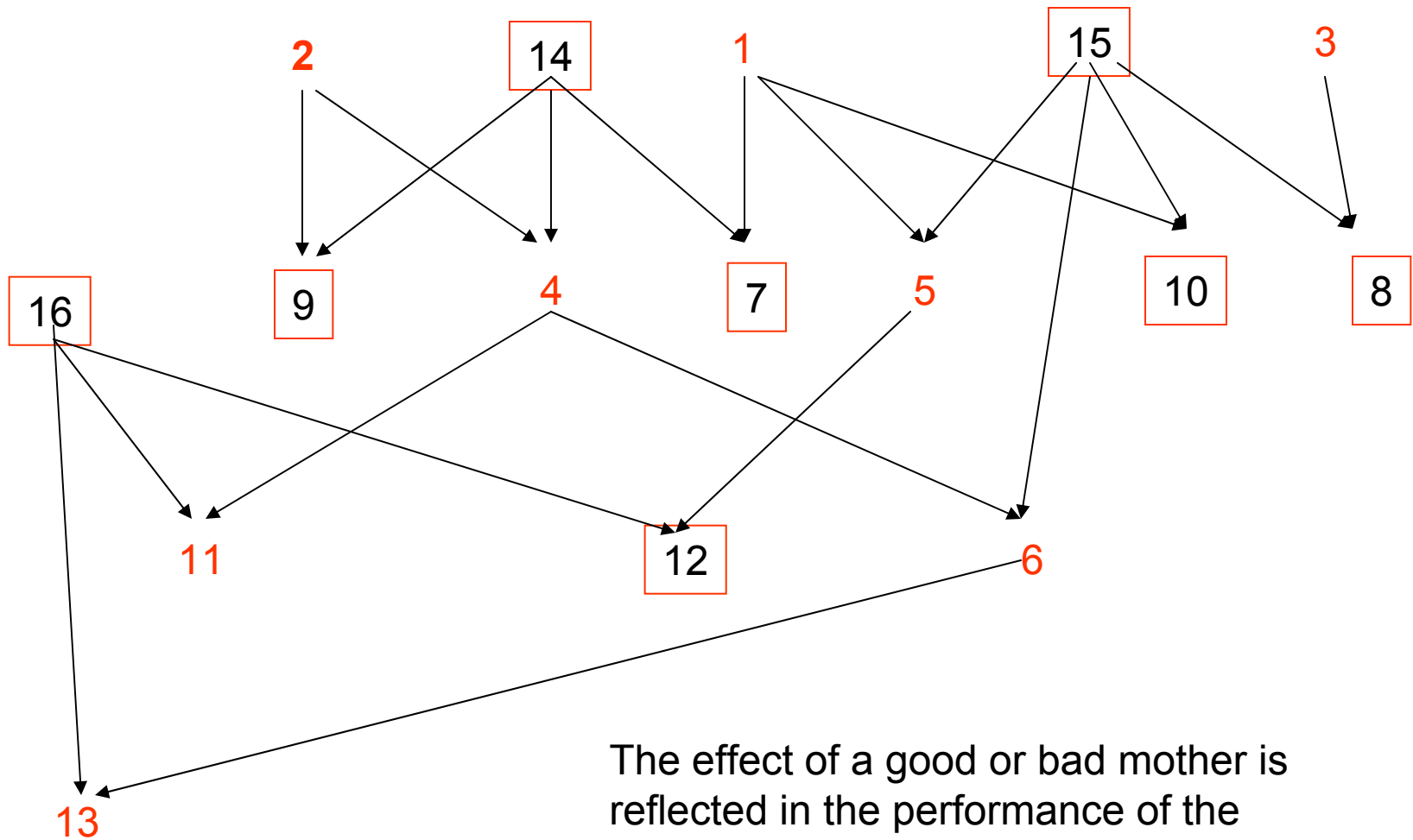
The are inherited and determined by additive effects in the mother

# Maternal Effects Example

Schaeffer Table 8.7

Animal	Sire	Dam	Year	Sex	Wean Wt
7	14	1	86	M	400
4	14	2	86	F	380
8	15	3	86	M	410
5	15	1	87	F	350
9	14	2	87	M	420
6	15	4	87	F	360
10	15	1	88	M	390
11	16	4	88	F	390
12	16	5	88	M	430
13	16	6	88	F	370

# Pedigree



The effect of a good or bad mother is reflected in the performance of the offspring

Year sex  
86 87 88 m

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

} Herd  
 Year  
 ← Sex

$$\mathbf{Y} = \begin{bmatrix} 400 \\ 380 \\ 410 \\ 350 \\ 420 \\ 360 \\ 390 \\ 390 \\ 430 \\ 370 \end{bmatrix}$$

# Animal

An	Sire	Dam
7	14	1
4	14	2
8	15	3
5	15	1
9	14	2
6	15	4
10	15	1
11	16	4
12	16	5
13	16	6

$Z_1 =$

	14	1	2	15	3	16	7	4	8	5	9	6	10	11	12	13
7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Animal 1 was the mother of animals 7, 5, 10

An	Sire	Dam
7	14	1
4	14	2
8	15	3
5	15	1
9	14	2
6	15	4
10	15	1
11	16	4
12	16	5
13	16	6

$Z_2 =$

	14	1	2	15	3	16	7	4	8	5	9	6	10	11	12	13
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0

# MME

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z}_1 & \mathbf{X}'\mathbf{Z}_2 \\ \mathbf{Z}_1'\mathbf{X} & \mathbf{Z}_1'\mathbf{Z}_1 + \mathbf{A}^{-1}\mathbf{k}_{11} & \mathbf{Z}_1'\mathbf{Z}_2 + \mathbf{A}^{-1}\mathbf{k}_{12} \\ \mathbf{Z}_2'\mathbf{X} & \mathbf{Z}_2'\mathbf{Z}_1 + \mathbf{A}^{-1}\mathbf{k}_{21} & \mathbf{Z}_2'\mathbf{Z}_2 + \mathbf{A}^{-1}\mathbf{k}_{22} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{a}} \\ \hat{\mathbf{m}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{Z}_1'\mathbf{y} \\ \mathbf{Z}_2'\mathbf{y} \end{bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \sigma_d^2 & \sigma_{dm} \\ \sigma_{dm} & \sigma_m^2 \end{bmatrix}^{-1} \sigma_e^2$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 2000 & -300 \\ -300 & 1200 \end{bmatrix}^{-1} 6500 = \begin{bmatrix} 3.3766 & .8441 \\ .8441 & 5.6288 \end{bmatrix}$$

```

proc iml;
start main;

y={400,
  380,
  410,
  350,
  420,
  360,
  390,
  390,
  430,
  370};

X={1 0 0 1,
  1 0 0 0,
  1 0 0 1,
  0 1 0 0,
  0 1 0 1,
  0 1 0 0,
  0 0 1 1,
  0 0 1 0,
  0 0 1 1,
  0 0 1 0};

Z1={0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0,
    0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0,
    0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0,
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0,
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1};

Z2={ 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0,
    0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0,
    0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0,
    0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0,
    0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0,
    0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0,
    0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0};

```

```

Ainv={2.5 .5 1 0 0 0 -1 -1 0 0 -1 0 0 0 0 0,
      .5 2.5 0 1 0 0 -1 0 0 -1 0 0 -1 0 0 0,
      1 0 2 0 0 0 0 -1 0 0 -1 0 0 0 0 0,
      0 1 0 3 .5 0 0 .5 -1 -1 0 -1 -1 0 0 0,
      0 0 0 .5 1.5 0 0 0 -1 0 0 0 0 0 0 0,
      0 0 0 0 0 2.5 0 .5 0 .5 0 .5 0 -1 -1 -1,
      -1 -1 0 0 0 0 2 0 0 0 0 0 0 0 0 0,
      -1 0 -1 .5 0 .5 0 3 0 0 0 -1 0 -1 0 0,
      0 0 0 -1 -1 0 0 0 2 0 0 0 0 0 0 0,
      0 -1 0 -1 0 .5 0 0 0 2.5 0 0 0 0 -1 0,
      -1 0 -1 0 0 0 0 0 0 0 2 0 0 0 0 0,
      0 0 0 -1 0 .5 0 -1 0 0 0 2.5 0 0 0 -1,
      0 -1 0 -1 0 0 0 0 0 0 0 0 2 0 0 0,
      0 0 0 0 0 -1 0 -1 0 0 0 0 0 2 0 0,
      0 0 0 0 0 -1 0 0 0 -1 0 0 0 0 2 0,
      0 0 0 0 0 -1 0 0 0 0 0 -1 0 0 0 2};

```

```

K11=3.3766;K12=.8441;K21=.8441;K22=5.6288;

```

```

LHS=((X`*X)||(X`*Z1)||(X`*Z2))

```

```

      //((Z1`*X)||(Z1`*Z1+AINV#K11)||(Z1`*Z2+AINV#K12))

```

```

      //((Z2`*X)||(Z2`*Z1+AINV#K21)||(Z2`*Z2+AINV#K22));

```

```

RHS=(X`*Y)/(Z1`*Y)/(Z2`*Y);

```

```

C=INV(LHS);

```

```

BU=C*RHS;

```

```

print BU ;

```

```

finish main;run;quit;

```

<b>B</b>		$\hat{a}$	Animal	$\hat{m}$	Animal
369.40	} Year	1.567	14	0.09	14
363.10		-2.43	12	-3.65	12
374.56		1.79	15	2.68	15
41.48	← Sex	-3.57	3	1.16	15
		0.07	16	0.10	3
		2.58	7	-0.38	16
		-1.34	4	-1.64	7
		2.67	8	1.55	4
		-1.65	5	0.61	8
		-3.28	9	-0.09	5
		3.15	6	1.16	9
		-1.26	10	1.00	6
		-5.59	11	-0.85	10
		4.13	12	0.36	11
		1.56	13	-0.52	12
		-0.16		0.43	13

Note that it is possible to estimate a maternal genetic effects for males.  
Why?

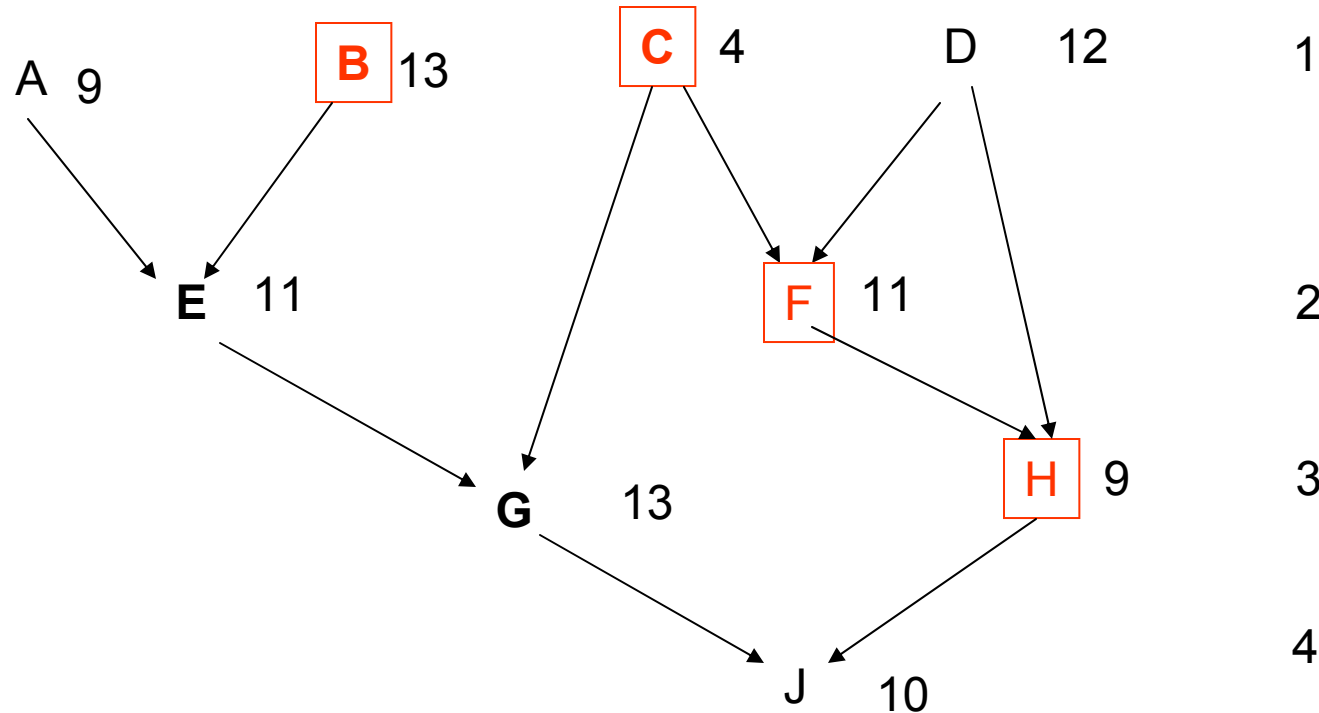
# What to do with the estimates in a breeding program

- Selection Index (to be covered later)
  - Give a weight to each effect and combine in an index

$$I_i = w_1 \hat{a}_i + w_2 \hat{m}_i$$

Weights are dependent on the economic impact of each trait on overall profits

# Lab Problem 14.1



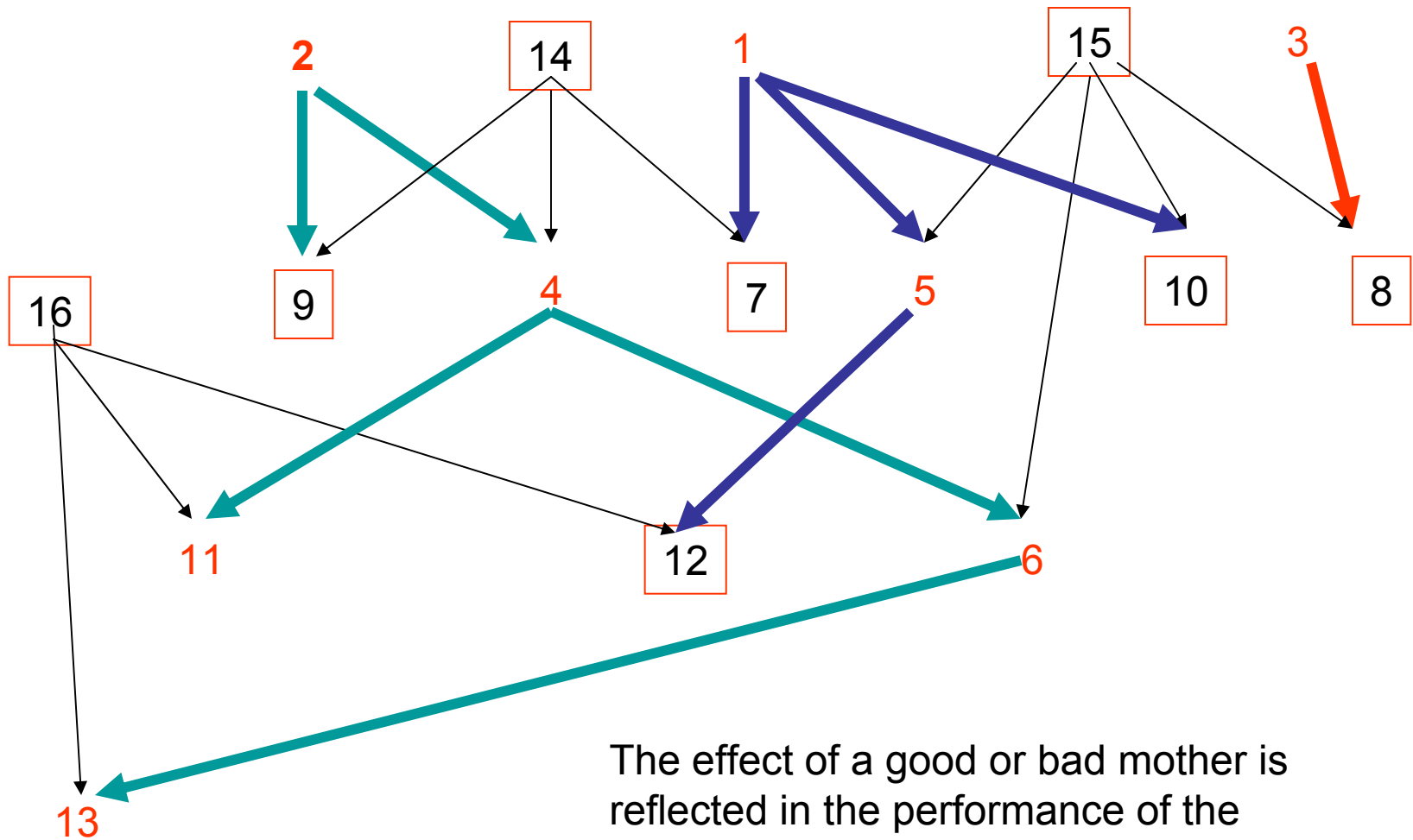
Find the best estimate of the environmental trend, genetic worth of each animal, Maternal Genetic Effect (males are in boxes), assume error variance as previously estimated in 6.2a and

$$\frac{\sigma_e^2}{\sigma_a^2} = 1 \quad \frac{\sigma_e^2}{\sigma_m^2} = .5 \quad \frac{\sigma_{a,m}}{\sigma_e^2} = -.25$$

# Cytoplasmic Effects

Follows Maternal Lines Only

# Pedigree



The effect of a good or bad mother is reflected in the performance of the offspring



# Cytoplasmic Effects Mixed Model

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{Z}_1\mathbf{d} + \mathbf{Z}_2\mathbf{c} + \mathbf{e}$$

Direct effect

Cytoplasmic

$$V \begin{bmatrix} \mathbf{d} \\ \mathbf{m} \\ \mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A}\sigma_d^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}\sigma_m^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\sigma_e^2 \end{bmatrix}$$

Assumes no cytoplasmic – nuclear gene interaction

Year sex  
86 87 88 m

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

} Herd  
 Year  
 ← Sex

$$\mathbf{Y} = \begin{bmatrix} 400 \\ 380 \\ 410 \\ 350 \\ 420 \\ 360 \\ 390 \\ 390 \\ 430 \\ 370 \end{bmatrix}$$

# Animal

An	Sire	Dam
7	14	1
4	14	2
8	15	3
5	15	1
9	14	2
6	15	4
10	15	1
11	16	4
12	16	5
13	16	6

$Z_1 =$

	14	1	2	15	3	16	7	4	8	5	9	6	10	11	12	13
7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Cytoplasmic

An	Sire	Dam
7	14	1
4	14	2
8	15	3
5	15	1
9	14	2
6	15	4
10	15	1
11	16	4
12	16	5
13	16	6

Animal 1 was the mother lineage of animals 7, 5, 10, 12

1 2 3 Maternal Lineages

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

# MME

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{Z}_1 & \mathbf{X}'\mathbf{Z}_2 \\ \mathbf{Z}_1'\mathbf{X} & \mathbf{Z}_1'\mathbf{Z}_1 + \mathbf{A}^{-1}k_{11} & \mathbf{Z}_1'\mathbf{Z}_2 \\ \mathbf{Z}_2'\mathbf{X} & \mathbf{Z}_2'\mathbf{Z}_1 & \mathbf{Z}_2'\mathbf{Z}_2 + \mathbf{I}k_{22} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{a}} \\ \hat{\mathbf{c}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{Z}_1'\mathbf{y} \\ \mathbf{Z}_2'\mathbf{y} \end{bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \sigma_d^2 & 0 \\ 0 & \sigma_c^2 \end{bmatrix}^{-1} \sigma_e^2 = \begin{bmatrix} \frac{\sigma_e^2}{\sigma_d^2} & 0 \\ 0 & \frac{\sigma_e^2}{\sigma_c^2} \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 2000 & 0 \\ 0 & 1200 \end{bmatrix}^{-1} 6500 = \begin{bmatrix} 3.25 & 0 \\ 0 & 5.42 \end{bmatrix}$$

```
proc iml;
start main;
```

```
y={400,
    380,
    410,
    350,
    420,
    360,
    390,
    390,
    430,
    370};
```

```
X={1 0 0 1,
    1 0 0 0,
    1 0 0 1,
    0 1 0 0,
    0 1 0 1,
    0 1 0 0,
    0 0 1 1,
    0 0 1 0,
    0 0 1 1,
    0 0 1 0};
```

```
Z1={0 0 0 0 0 0 1 0 0 0 0 0 0 0 0,
     0 0 0 0 0 0 0 1 0 0 0 0 0 0 0,
     0 0 0 0 0 0 0 0 1 0 0 0 0 0 0,
     0 0 0 0 0 0 0 0 0 1 0 0 0 0 0,
     0 0 0 0 0 0 0 0 0 0 1 0 0 0 0,
     0 0 0 0 0 0 0 0 0 0 0 1 0 0 0,
     0 0 0 0 0 0 0 0 0 0 0 0 1 0 0,
     0 0 0 0 0 0 0 0 0 0 0 0 0 1 0,
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 1};
```

```
Z2={ 1 0 0,
     0 1 0,
     0 0 1,
     1 0 0,
     0 1 0,
     0 0 0,
     1 0 0,
     0 0 0,
     0 0 0,
     0 0 0,
     0 0 0};
```

```
I={1 0 0,
   0 1 0,
   0 0 1};
```

```

Ainv={2.5 .5 1 0 0 0 -1 -1 0 0 -1 0 0 0 0 0,
      .5 2.5 0 1 0 0 -1 0 0 -1 0 0 -1 0 0 0,
      1 0 2 0 0 0 0 -1 0 0 -1 0 0 0 0 0,
      0 1 0 3 .5 0 0 .5 -1 -1 0 -1 -1 0 0 0,
      0 0 0 .5 1.5 0 0 0 -1 0 0 0 0 0 0 0,
      0 0 0 0 0 2.5 0 .5 0 .5 0 .5 0 -1 -1 -1,
      -1 -1 0 0 0 0 2 0 0 0 0 0 0 0 0 0,
      -1 0 -1 .5 0 .5 0 3 0 0 0 -1 0 -1 0 0,
      0 0 0 -1 -1 0 0 0 2 0 0 0 0 0 0 0,
      0 -1 0 -1 0 .5 0 0 0 2.5 0 0 0 0 -1 0,
      -1 0 -1 0 0 0 0 0 0 0 2 0 0 0 0 0,
      0 0 0 -1 0 .5 0 -1 0 0 0 2.5 0 0 0 -1,
      0 -1 0 -1 0 0 0 0 0 0 0 0 2 0 0 0,
      0 0 0 0 0 -1 0 -1 0 0 0 0 0 2 0 0,
      0 0 0 0 0 -1 0 0 0 -1 0 0 0 0 2 0,
      0 0 0 0 0 -1 0 0 0 0 0 -1 0 0 0 2};

```

**K11=3.25;K12=0;K21=0;K22=5.42;**

LHS=((X`\*X)||X`\*Z1)||X`\*Z2))

//((Z1`\*X)||Z1`\*Z1+AINV#K11)||Z1`\*Z2))

//((Z2`\*X)||Z2`\*Z1)||Z2`\*Z2+I#K22));

RHS=(X`\*Y)//Z1`\*Y)//Z2`\*Y);

C=INV(LHS);

BU=C\*RHS;

RMSE=(Y`\*Y-BU`\*RHS)#(1/6);

print BU RMSE;

finish main;run;quit;

<b>B</b>		$\hat{a}$	Animal	$\hat{c}$	Animal
368.15	}	1.55	14	-2.81	1
361.75		-3.45	12	2.73	2
373.55		2.53	15	0.08	3
42.94	←	-3.40	3		
		0.06	16		
		2.70	7		
		-1.92	4		
		3.17	8		
		-1.60	5		
		-3.20	9		
		3.44	6		
		-1.10	10		
		6.12	11		
		4.37	12		
		1.95	13		
		-0.14			

# What to do with the estimates in a breeding program

- Selection Index

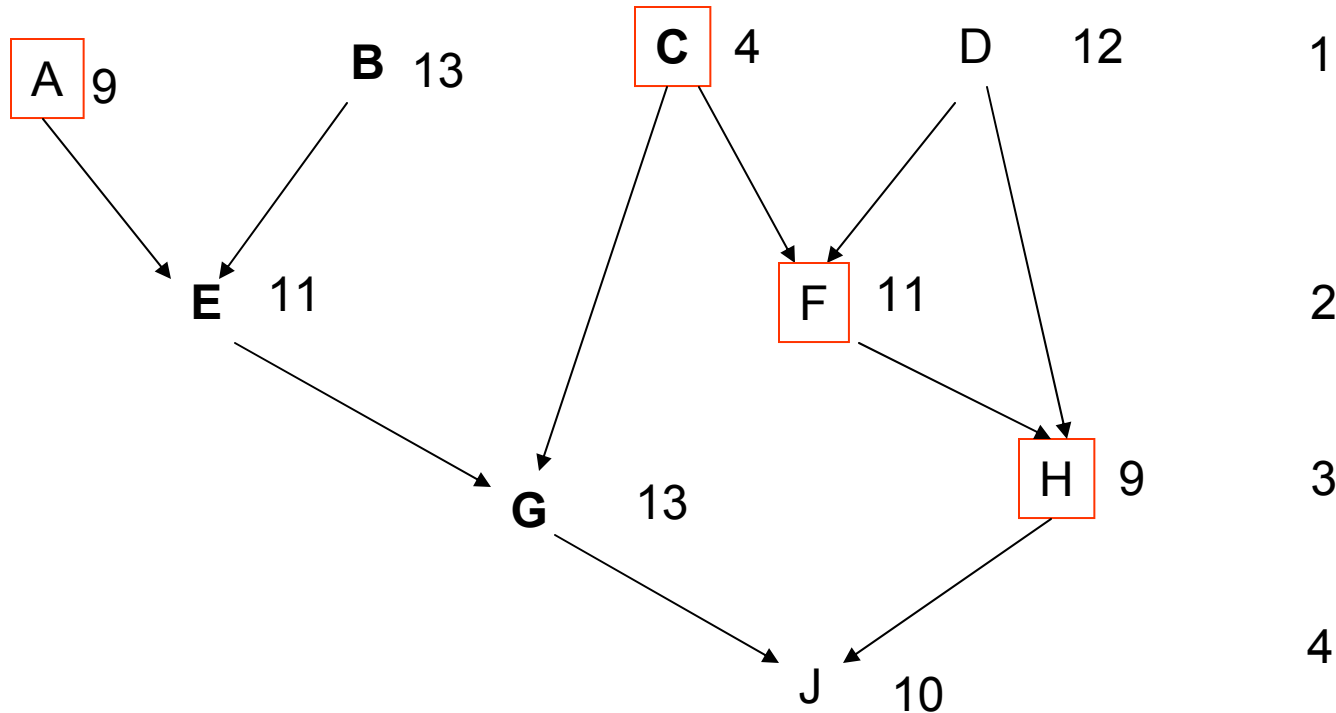
- Give a weight to each effect and combine in an index

$$I_i = w_1 \hat{a}_i + w_2 \hat{c}_i$$

Weights are dependent on the economic impact of each trait on overall profits.

Economic impact of cytoplasmic effect changes with the time horizon. Over a large number of generation could be a very substantial effect

# Lab Problem 14.2



Find the best estimate of the environmental trend, genetic worth of each animal, and cyto-genetic effects. Assume error variance as previously estimated in 6.2a and

$$\frac{\sigma_e^2}{\sigma_a^2} = 1 \qquad \frac{\sigma_e^2}{\sigma_c^2} = .5$$